※ 注意：請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。

複選題(答案可能有一個或是多個)

1. (5 分) Laplace transform $L\{-2\cos 2t + 3\sin 2t\}$ = (A) $\frac{2s-6}{s^2+4}$; (B) $\frac{2s-6}{s^2-4}$; (C) $\frac{-2s+6}{s^2+4}$; (D) $\frac{-2s+6}{s^2-4}$; (E) 以上皆非。

2. (5 分) Fourier transform $F\{\exp(-x^2/4p^2)\}$ = (A) $2\sqrt{\pi}p\exp(-4p^2\alpha^2)$; (B) $2\sqrt{\pi}p\exp(-p^2\alpha^2/4)$; (C) $2\sqrt{\pi}p\exp(-\alpha^2/p^2)$; (D) $2\sqrt{\pi}p\exp(-p^2\alpha^2)$; (E) 以上皆非。

3. (7 分) 微分方程式 $(x^2+1)y''+xy'-y=0$ 的級數解為
(A) $1+x$; (B) $1+\frac{1}{2}x^2+\sum_{n=2}^{\infty}(-1)^{n-1}\frac{1\cdot 3\cdot 5\cdots(2n-3)}{2^n n!}x^{2n}$, $|x|<1$; (C) $1+\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1\cdot 3\cdot 5\cdots(2n-1)}{2^n n!}x^{2n}$, $|x|<1$; (D) x; (E)以上皆非。

4 (8 分) 針對以下的微分方程組 $\frac{dx}{dt}=-4x+y+z$，$\frac{dy}{dt}=x+5y-z$，$\frac{dz}{dt}=y-3z$ 其一般解為 (A) $x=C_1e^{-3t}+10C_2e^{-4t}+C_3e^{5t}$; (B) $y=C_1e^{-3t}-C_2e^{-4t}+8C_3e^{5t}$; (C) $z=C_1e^{-3t}+C_2e^{-4t}+C_3e^{5t}$; (D) $z=-C_2e^{-4t}+8C_3e^{5t}$; (E) 以上皆非。

5.(5 分) The *trace* of an square $n\times n$ matrix

$$A=\begin{pmatrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,n} \end{pmatrix}$$

is defined to be the sum of its diagonal elements, i.e.,

$$\text{trace } A = a_{1,1}+\cdots+a_{n,n}$$

Which of the following statements about the trace of a matrix are true?
(A)trace is linear, i.e., $\text{trace}(A+B)=\text{trace } A+\text{trace } B$ and $\text{trace}(cA)=c\,\text{trace } A$ for any $n\times n$ matrices A, B and any scalar c.
(B)If A is an $m\times n$ matrix and B an $n\times m$ matrix, then $\text{trace}(AB)=\text{trace}(BA)$.
(C)If A, B, and C are $n\times n$ matrices, then $\text{trace}(ABC)=\text{trace}(CBA)$.
(D)If A is an $n\times n$ matrix such that $A^2=A$, then trace A is a nonnegative integer.
(E)None of the above are true.

6.(5 分) Suppose that A is a real $n\times n$ square matrix such that $A^2+\alpha A+\beta I=0$ for some real numbers α and β, where I is the $n\times n$ identity matrix and 0 the $n\times n$ zero matrix. Which of the following statements are true?
(A)A necessary condition for A to have a (real) eigenvalue is that $\alpha^2<4\beta$.
(B)A sufficient condition for A to have a (real) eigenvalue is that $\alpha^2<4\beta$.
(C)A necessary condition for $\alpha^2\geq 4\beta$ is that A has a (real) eigenvalue.
(D)A sufficient condition for $\alpha^2\geq 4\beta$ is that A has a (real) eigenvalue.
(E)None of the above are true.

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7.(5 分) Let $\mathbf{V}$ be a finite-dimensional vector space, and let $\mathbf{U}_1, \mathbf{U}_2$, and $\mathbf{U}_3$ be its subspaces. Which of the following statements are true?

(A) $\dim(\mathbf{U}_1 \cup \mathbf{U}_2) = \dim\mathbf{U}_1 + \dim\mathbf{U}_2$.

(B) $\dim(\mathbf{U}_1 + \mathbf{U}_2) = \dim\mathbf{U}_1 + \dim\mathbf{U}_2 - \dim(\mathbf{U}_1 \cap \mathbf{U}_2)$.

(C) $\dim(\mathbf{U}_1 \cup \mathbf{U}_2 \cup \mathbf{U}_3) = \dim\mathbf{U}_1 + \dim\mathbf{U}_2 + \dim\mathbf{U}_3$.

(D) $\dim(\mathbf{U}_1 + \mathbf{U}_2 + \mathbf{U}_3) = \dim\mathbf{U}_1 + \dim\mathbf{U}_2 + \dim\mathbf{U}_3 - \dim(\mathbf{U}_1 \cap \mathbf{U}_2) - \dim(\mathbf{U}_1 \cap \mathbf{U}_3) - \dim(\mathbf{U}_2 \cap \mathbf{U}_3) + \dim(\mathbf{U}_1 \cap \mathbf{U}_2 \cap \mathbf{U}_3)$

(E) None of the above are true.

8.(5 分) Which of the following subsets of $\mathbf{R}^3$ are also subspaces of $\mathbf{R}^3$?

(A) $\{(x_1, x_2, x_3) \in \mathbf{R}^3 : x_1 + 2x_2 + 3x_3 = 0\}$.

(B) $\{(x_1, x_2, x_3) \in \mathbf{R}^3 : x_1 + 2x_2 + 3x_3 = 4\}$.

(C) $\{(x_1, x_2, x_3) \in \mathbf{R}^3 : x_1 x_2 x_3 = 0\}$.

(D) $\{(x_1, x_2, x_3) \in \mathbf{R}^3 : x_1 = 5x_3\}$.

(E) None of the above are subspaces of $\mathbf{R}^3$.

9.(5 分) Which of the following statements are true?

(A) If T is a linear transformation from $\mathbf{R}^4$ to $\mathbf{R}^2$ such that null $T = \{(x_1, x_2, x_3, x_4) \in \mathbf{R}^4 : x_1 = 5x_2 \text{ and } x_3 = 7x_4\}$, then T is surjective.

(B) There does not exist a linear transformation from $\mathbf{R}^5$ to $\mathbf{R}^2$ whose null space equals $\{(x_1, x_2, x_3, x_4, x_5) \in \mathbf{R}^5 : x_1 = 3x_3 \text{ and } x_3 = x_4 = x_5\}$.

(C) Suppose that $\mathbf{V}$ and $\mathbf{W}$ are finite-dimensional vector spaces and that $\mathbf{U}$ is a subspace of $\mathbf{V}$. Then there exists a linear transformation T from $\mathbf{V}$ to $\mathbf{W}$ such that null $T = \mathbf{U}$ if and only if $\dim\mathbf{U} \ge \dim\mathbf{V} - \dim\mathbf{W}$.

(D) Suppose that T is an $n \times n$ square matrix. Then T is a scalar multiple of the $n \times n$ identity matrix if and only if $ST = TS$ for every $n \times n$ square matrix S.

(E) None of the above are true.

10.(5 分) The differential equation $x^2 y'' + xy' + (x^2 - \frac{1}{4}) = 0$ is singular about x=0. We will use the method of Freobenius to obtain two linearly independent series solutions about x=0. Two indicial roots are r_1 and r_2 with $r_1 > r_2$. The solution has the form as $y = C_1 x^{r_2} \sum_{n=0}^{\infty} P_{2n+1} x^{2n+1} + C_2 x^{r_2} \sum_{n=0}^{\infty} Q_{2n} x^{2n}$.

(A) $r_1 + r_2 = 0$, (B) $r_1 - r_2 = 2$, (C) $P_1 = -\frac{1}{6}$, (D) $\frac{Q_{2n}}{P_{2n+1}} = 2n+1$, (E) none of above.

11.(5 分) The following electromotive force $E(t) = \begin{cases} 50, & 0 \le t \le 20 \\ 0, & t > 20 \end{cases}$ Volt,

is applied to an LR series circuit with inductance of 10 Henry and resistance of 2 Ω with

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initial condition $i(0)=0$.

(A) $i(20)=25-25e^{-4}$ A, (B) $i(10)=25-25e^{-50}$ A, (C) $i(30)=25\left(e^{-2}-e^{-6}\right)$ A, (D) $i(\infty)=0$ A, (E) none of above.

12.(5 分) Solve the differential equation $(2y^2+3x)dx+(2xy)dy=0$.

(A) With given initial condition $y(1)=2$, then $y(-1)=\sqrt{6}$. (B) With given initial condition $y(1)=-2$, then $y(-1)=\sqrt{6}$, (C) With given initial condition $y(1)=2$, then $y(-1)=-\sqrt{6}$, (D) With given initial condition $y(1)=-2$, then $y(-1)=-\sqrt{6}$, (E) none of above.

13.(5 分) Solve the following differential equations system $x''-y-2x=-e^t$ and $y''-3x-4y=-7e^t$ with boundary conditions: $x(0)=\frac{4}{3}$ and $y(0)=2$.

(A) $3x(1)+y(1)=6e+2e^5$ (B) $-3x(-1)+y(-1)=2e^{-1}$ (C) $-3x(1)+y(-1)=4e^{-1}$ (D) $3x(-1)+y(1)=2\cosh(5)$ (E) none of above.

14.(5 分) For a series RLC circuit with driving voltage E(t)=300 Volt, L=5/3 Henry, R=10 Ω and C=1/30 F. q(t) is the charge on the capacitor. i(t) is the current flow through the RLC circuit. With given initial condition as q(0)=0 C and i(0)=0A. Let $q(t)=A-e^{-Bt}(C_1\cos(Dt)+C_2\sin(Et))$.

(A) $q(\infty)=30C$. (B) The oscillating frequency is 3Hz (C) $B+D=5$ (D) $C_1=C_2$ (E) $A=C_2$.

15. (5 分) For any $n\times n$ matrix H, if there is an orthogonal basis for R^n consisting of eigenvectors of A, then

(A) $H^T=H$

(B) H is an orthogonal matrix

(C) Hv is orthogonal to v for every eigenvector v of A

(D) P^THP is a diagonal matrix for every orthogonal $n\times n$ matrix P

(E) none of the preceding statements are true

16. (5 分) Suppose that v, w, and z are vectors in R^n such that v is orthogonal to z and z is orthogonal to w. Then

(A) For any orthogonal $n\times n$ matrix H, we have that Hz is orthogonal to both v nd w

(B) $v+w$ is orthogonal to z

(C) v is orthogonal to w

(D) For any orthogonal $n\times n$ matrix H, we have that Hv is orthogonal to z

(E) none of the preceding statements are true

17. (5 分) For any $n \times n$ symmetric matrix H,
   (A) Eigenvectors corresponding to distinct eigenvalues of H are orthogonal
   (B) Distinct eigenvectors of H are orthogonal
   (C) Distinct columns of H are orthogonal
   (D) Distinct rows of H are orthogonal
   (E) none of the preceding statements are true

18. (5 分) For any vector space W,
   (A) If W is finite-dimensional, then W is a subspace of R^n for some positive integer n
   (B) If W is finite-dimensional, then no infinite subset of W is linearly independent
   (C) If W is a function space, then W must be infinite-dimensional
   (D) If W is infinite-dimensional, then every infinite subset of W is linearly independent
   (E) none of the preceding statements are true

19. (5 分) Let H be an arbitrary $n \times n$ matrix. Then
   (A) The row space of H is contained in the column space
   (B) The row space of H equals the column space of H
   (C) The row space of H has the same dimension as the column space of H
   (D) The row space of H equals the null space of H
   (E) None of the preceding statements is true