1) Apply Laplace transform to solve the initial value problem. (20%)

$$\frac{dy_1}{dt} = 5y_1 + 5y_2 - 15\cos t + 27\sin t$$
$$\frac{dy_2}{dt} = -10y_1 - 5y_2 - 150\sin t$$
$$y_1(0) = 2$$
$$y_2(0) = 2$$

2) Find the eigenvalues (5%) and eigenfunctions (5%) of the differential equation below and also check the orthogonality between the eigenfunctions (5%).

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + (\lambda + 1)y = 0$$
$$y(0) = 0$$
$$y(1) = 0$$

3) Solve the differential equation. (15%)

$$x^2\frac{d^2y}{dx^2} - 7x\frac{dy}{dx} + 16y = 0$$

4) Please solve the boundary value problem. (20%)

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} \text{ for } 0 < x < \pi,\ t > 0,$$
$$y(0,t) = y(\pi,t) = 0 \text{ for } t \geq 0,$$
$$y(x,0) = 0 \text{ for } 0 \leq x \leq \pi,$$
$$\frac{\partial y}{\partial t}(x,0) = x(1 + \cos(x)), \text{ for } 0 \leq x \leq \pi$$

5) Please integrate the following functions around C counterclockwise or as indicated.

(a) $f(z) = \dfrac{2z^3 - 3}{z(z-1-i)^2}$, C consists of $|z| = 2$ (counterclockwise) and $|z| = 1$ (clockwise). (10%)

(b) $f(z) = z^{-2}\tan \pi z$, C any contour enclosing 0. (10%)

6) Please find the Fourier series of $f(x)$. (10%)

$$f(x)\begin{cases} x, & if \quad -\pi/2 < x < \pi/2 \\ \pi - x, & if \quad \pi/2 < x < 3\pi/2 \end{cases}$$

試題隨卷繳回