

※ 注意：請於試卷上依序作答，並應註明作答之大題及其題號。

1. We are trying to develop an energy harvesting device to collect energy from stepping. The system is illustrated as in Figure 1, where F represents the step force, x represents the downward displacement of the step, m is the mass of the pad, b_1 and k_1 are the damping ratio and the spring constant of the pad, respectively. The rack and pinning has a pitch of $p = 2\pi r_2/100$, where r_2 is the pitch radius of the pinning. The electrical generator has a back emf constant equal to the torque constant, K_B . Assume the battery behaves like a capacitor, C_2 , with internal resistant R_2 . i_2 and v_o are the current into the battery and voltage across the battery.

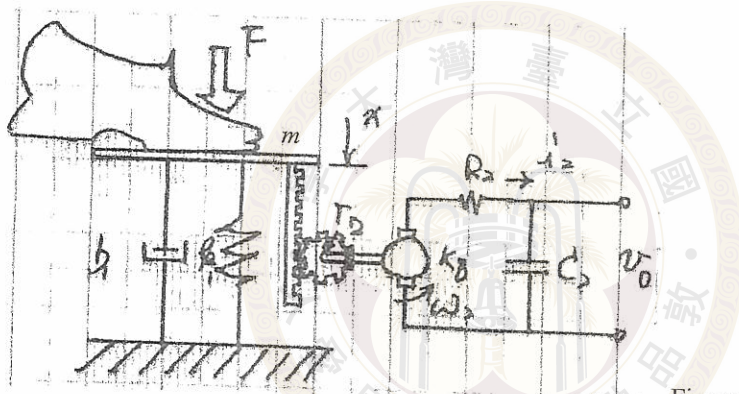


Figure 1

- What is the relationship between the time derivative of the displacement, \dot{x} , and the rotor speed, ω_2 ? (5%)
- Let ω_2 be the rotor speed of the generator. Derive the transfer function from the rotor speed Ω_2 to the output voltage V_o . (5%)
- Now the step force is balanced by three forces: the spring force, the damping force, and the reaction force from the rack and pinning; neglect the inertia of the rack and pinning; write down the force balance equation governing the pad movement. (5%)
- Derive the transfer function of the system from the input step force, $F(s)$, to the output voltage, $V_o(s)$, across the battery voltage. (10%)

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2. Consider a higher order system $G(s) = \frac{0.125s^2 + 2.51s + 10.1}{0.0125s^3 + 0.255s^2 + 0.15s + 1}$, the step response of the open-loop system is very oscillatory (see Figure 2). System identification reveals that there is a pair of complex poles at $-0.2 \pm j1.99$.

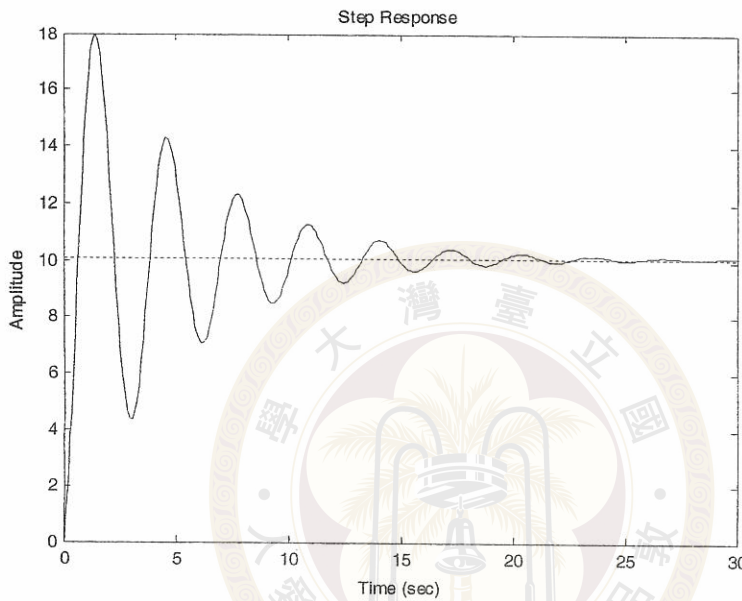


Figure 2

- Use partial fraction expansion to decompose the system dynamics into fast and slow dynamics. (5%)
- Can you use a proportional control to tune up the closed-loop damping ratio and suppress this oscillation, say to $\zeta = 0.8$? (5%)
- What is the smallest order control required if we were to suppress the steady state error to arbitrary small? (5%)
- How would the system behave if we use only proportional control and crank up the gain real high? (10%)

3. Sketch Bode plots of the following systems (please specify the slopes and significant angles):

a). $G(s) = \frac{s+10}{s(s+1)(s+100)}$. (5%)

b). $G(s) = \frac{s-2}{s(s+50)}$. (5%)

c). $G(s) = \frac{s+10}{s(s-1)}$. (5%)

4. Consider the closed-loop system in Figure 4, where $G(s) = \frac{s-b}{s(s+a)}$ with $a > b > 0$

a). Using **root-locus**, show that the closed-loop system cannot be stabilized by a proportional (P) controller $K(s) = K_p$ with $K_p \in [0, \infty]$. (5%)

b). Using **root-locus**, show that the system can be stabilized by a proportional-derivative (PD) controller $K(s) = K_p + K_D s = K_D(s+c)$. (Note: in practice, we normally set $K(s) = \frac{K_p + K_D s}{s+\tau}$ with a large τ to approximate $K(s) = K_p + K_D s$.) (5%)

c). Suppose $G(s) = \frac{s-1}{s(s+10)}$, it is known that the closed-loop poles can be arbitrarily assigned using a first-order controller $K(s) = \frac{e_1 s + e_0}{s+d_0}$. Please find a suitable controller $K(s)$ such that the closed-loop poles are all located at $s = -2$. (5%)

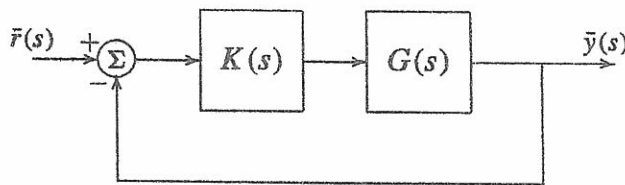


Figure 4

5. Refer to Figure 4 with $K(s)G(s) = \frac{\alpha(s^2 + 4s + 4)}{s^3}$,

a). Draw the Nyquist plot of the system when $\alpha = 1$. (5%)

b). Find the gain margin and phase margin and phase margin of the system when $\alpha = 1$. (5%)

c). State Nyquist Criterion, and use **Nyquist Criterion** to decide the range of α such that the closed-loop system is stable. (10%)