1.

(a)(10%)The velocity components of a flow field is given as

$$u=2x-y, \quad v=-x+3z, \quad w=3y+z.$$

If the temperature of the flow field is

$$T=x^2+2yz-xy-5t,$$

find the time-rate-of-change of temperature, dT/dt, at position (3, 0, 1).

(b)(5%)A velocity field is given as

$$u=x^2+y^2, \quad v=2xy, \quad w=-4z^2.$$

On what plane this velocity field can be used to describe an incompressible fluid flow?

(c) (10%)Given a velocity of a two-dimensional flow as

$$u=x, \quad v=-y,$$

is it possible to find the velocity potential Φ for the flow field? If it is possible, find it.

2.

(a)(5%)Due to temperature stratification or sediment deposition etc., the fluid density in a reservoir usually is not a constant. Suppose the fluid density is given as

$$\rho=\rho_0(1-\alpha z/H), \quad -H\leq z\leq 0,$$

where ρ_0 is the density on the free surface, and α is a constant, while H is the depth of the reservoir shown in Fig. 2(a). Find the static pressure distribution P(z) provided that P(0)=0.

(b)(15%)A cylindrical tank with diameter d contains water of density ρ and has a hemispherical bottom and a hemispherical cover with an opening on the top as shown in Fig. 2(b). Determine the magnitude, line of action, and direction of the force of the water on the curved bottom.

(c)(5%)For the Piezometer tube indicated in Fig. 2(c), let P_{atm} be zero, there is no surface tension effect, but given a vertical upward acceleration a_z, find P_A.

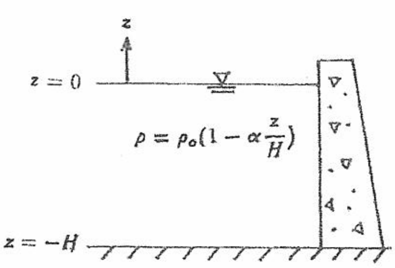

Fig. 2(a)

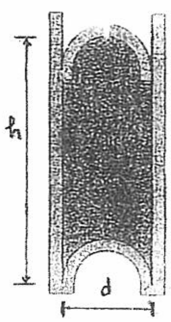

Fig. 2(b)

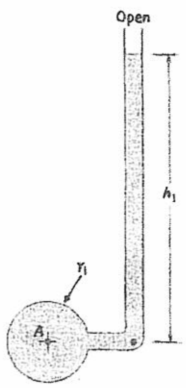

Fig. 2(c)

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3.

The following is a sluice gate experiment. Water is stored behind the gate initially. Then when gate opens there is water flowing to down stream. The inflow is larger than flow through the gate initially. But gradually, the equilibrium condition is reached. The flow condition around the gate is shown in Fig. 3. Neglect energy dissipation for this problem.

(a)(5%) Explain why the free surface first increases then decrease.

(b)(10%) If the contraction coefficient is $C_c = \frac{y_1}{y_g}$, prove that the flow rate per unit flume width is

$$q = C_c y_g y_o \sqrt{\frac{2g}{y_o + y_1}}$$

(c)

(i)(5%) The flume width b is 0.5m, the inflow rate Q is 0.1cms. What is the critical depth (i.e. the water depth for Froude number equals one)?

(ii)(5%) Assuming the contraction coefficient is 0.6, if we want to see the hydraulic jump phenomenon, what should the gate opening y_g be?

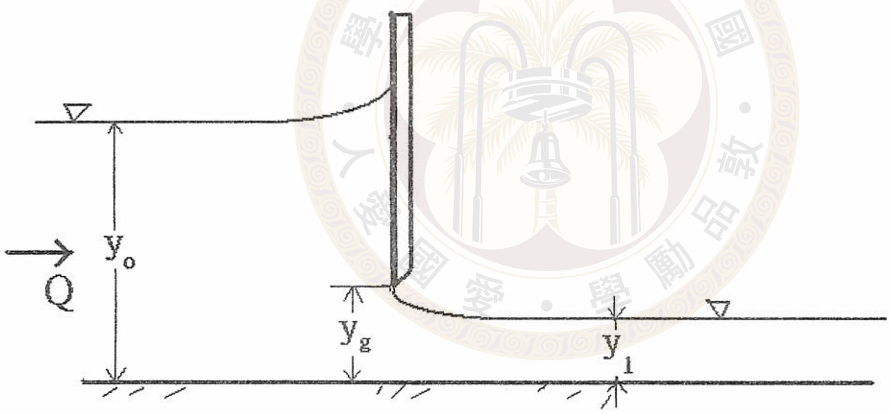

Fig. 3

4.

In Fig. 4(a), a flow rate of 60π L/min of oil of relative density 0.92 exists in pipeline AB. A small pipe CD is filled with mercury. The viscosity of the oil μ_{oil} is the same as that of water and the viscosity of mercury μ_{Hg} is 8 time that of water. The diameter of large pipe AB is 25mm and the small pipe CD is 10mm. Note: 1 L = 10^{-3} m, $\mu_{water} = 10^3$ kg/(s · m), $\rho_{water} = 10^3$ kg/m^3 .

(a)(3%) Calculate the Reynolds number based on pipe diameter in large pipe AB. Is it laminar flow? Why? Hint: $Re_{critical} \fallingdotseq 2100$.

(b)(14%) If the large pipe has a roughness of 0.01cm, what is the pressure gradient between point A and B? What is the direction of the pressure gradient? Hint: refer to Fig. 4(b).

(c)(8%)Using result from above, calculate H in the small pipe. If the flow starts and the situation in figure is reached in 1 second without any oscillation of interface in small pipe CD. Calculate the Reynolds number of mercury based pipe diameter in small pipe CD. Is it laminar flow?

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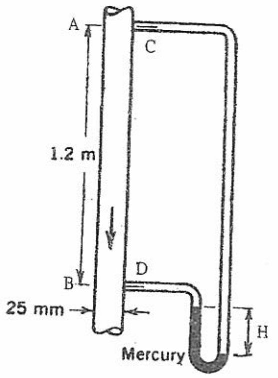
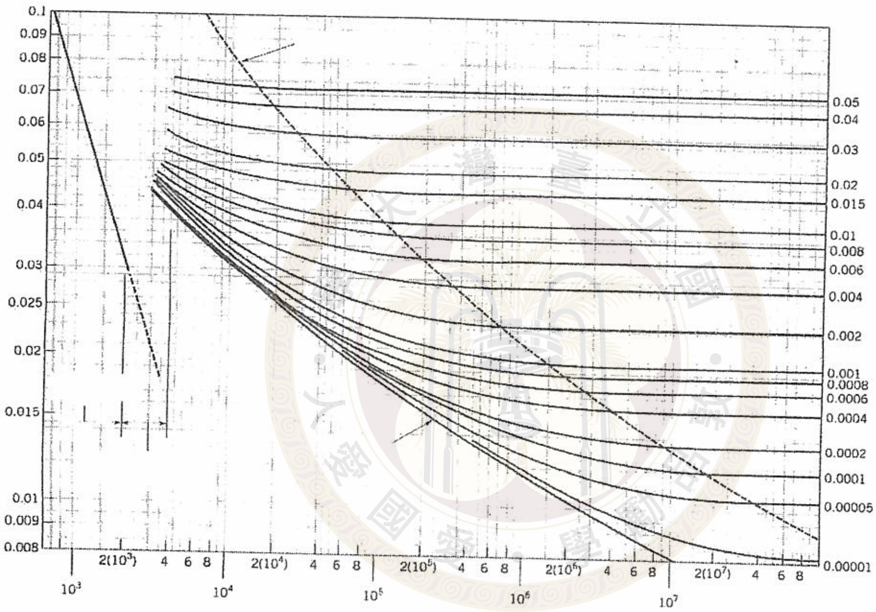

A
C
1.2 m
B
D
25 mm
H
Mercury

Fig. 4(a)


0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.025
0.02
0.015
0.01
0.009
0.008
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0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.00005
0.00001

Fig. 4(b)