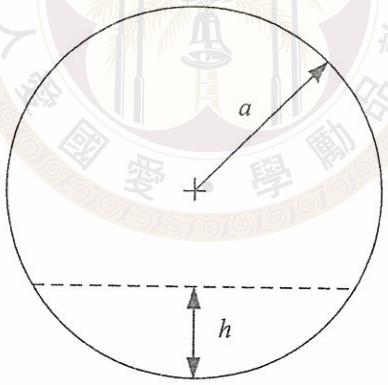
1. **(15%)** Consider a matrix $\mathbf{M}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Under what condition is this matrix singular?

2. **(15%)** Find the derivative $\frac{dy}{dx}$ of the following function: $2x^3+x^2y+y^3=1$. (**hint:** be aware that y is an implicit function of x).

3. **(20%)** The hyperbolic sine and cosine are defined as: $\sinh x=\frac{e^x-e^{-x}}{2}$ and $\cosh x=\frac{e^x+e^{-x}}{2}$. Prove the following identity: $\sinh(x+y)=\sinh x\cosh y+\cosh x\sinh y$.

4. **(20%)** The volume of liquid in a tank is given by: $V=\frac{1}{3}\pi h^2(3a-h)$ in which h is the depth from the bottom of the tank (Figure 1). If the tank is being filled with water at a rate of Q liters per minute, determine an expression for the rate of rise of the level of liquid in the tank for a given value of h.



(Figure 1)

5. **(15%)** Find the perpendicular distance from the point (1, -1, 2) to the plane $2x+2y-z+4=0$.

6. **(15%)** Integrate the following expression: $\int\sqrt{x}\ln x\,dx$.

試題隨卷繳回