※ 注意：請於「非選擇作答區」作答，選擇題不需列推演過程，計算或證明題須列過程，請務必標示題號。

1. (4%) Let S={1, 2, 3, 4, 5, 6} and the probability p(s) = 1/6 for all $s \in S$. Given four events E1={ 1, 2, 3}, E2={2, 4}, E3={4}, and E4={2, 3, 4, 5}, which of the following statement are correct
   A. E1 and E3 are mutually independent,
   B. E2 and E3 are mutually independent,
   C. E1 and E4 are mutually independent,
   D. E2 and E4 are mutually independent, or
   E. None of above

2. (4%) A and B are playing a game as follow. First, A and B pick their own numbers uniformly from {1, 2, 3, 4, 5} and {1, 3, 8}, respectively. Assume that A's number (say a1) is smaller than or equal B's number (say b1), then A gets one dollar. Whoever gets one dollar (in this case, A) can pick a new number, say a2, for the next run using his uniform distribution. Whoever does not get one dollar (in this case, B) will use a new number b2=b1-a1 for the next run. The same rule applies to B. (Note that it is possible that both get one dollar in the same run). Let A and B play this game for an infinite number of time, *what is the ratio of A's money and B's money at the end of the game?*
   A. 3:5
   B. 3:4
   C. 5:3
   D. 5:4
   E. Non of above

3. (4%) Let X1, X2, ... and Xn are independent and identical random variables with a CDF F(x). Let Z=min(X1, X2, ..., Xn). Then the PDF of Z can be represented as
   A. $(dF(x)/dx)^n$
   B. $F(x)^{n-1}dF(x)/dx$
   C. $n*F(x)^{n-1}dF(x)/dx$
   D. $n*[1-F(x)]^{n-1}dF(x)/dx$
   E. None of above

4. (4%) Let $P(Y=y|X=x)=x^y/y!*e^{-x}$ for y=0, 1, 2, ... and X is a zero mean Gaussian random variable with variance = 1. Then E[Y] =
   A. $1/\sqrt{2\pi}$
   B. $1/2\pi$
   C. $\sqrt{2\pi}$
   D. 1/2
   E. None of above

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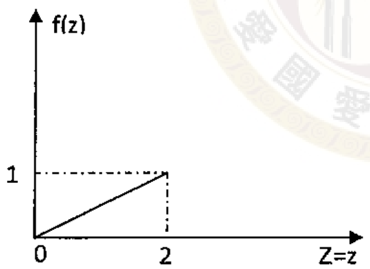
5. (4%) The moment generating function M(s) of a Poisson random variable with mean = α is
   - A. $M(s) = \alpha e^{\alpha s)}$
   - B. $M(s) = e^{\alpha(e^{s}-1)}$
   - C. $M(s) = \alpha!\, e^{\alpha}(1 + e^{s})$
   - D. $M(s) = e^{\alpha s+s+1}$
   - E. None of above

6. (8%) X is a continuous random variable and is said to be memoryless if Pr(X=t1) = Pr(X =t1+t2|X>t2). Please find the PDF of such an X and show that it is indeed memoryless.

7. X is a random with a PDF such that $\Pr(X>x)=(x/c)^{-k}$ for all $x \geq c$, where c and k are both constants. Please find the mean and variance of X (7%). Please find a random variable X and its PDF such that X has a finite mean but an infinite variance (3%).

8. Let X have a CDF F(x) and Y=F(X). Please find the PDF of Y (4%) and calculate Pr(Y > 0.6) (2%). If we have a uniform random variable generator, please show how to generate a random variable Z such that Z's pdf f(Z=z) is as shown in the figure below (6%).



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9. (5%) For the given equation $\int_0^t f(\tau)f(t-\tau) = \Lambda(t) = \begin{cases} 1-|t|, \text{for } |t| \le 1 \\ 0, \text{otherwise} \end{cases}$, please choose the **correct one answer**

(A) $f(t) = \begin{cases} 1/2, \text{for } |t| < 1/2 \\ 0, \text{otherwise} \end{cases}$; (B) $f(t) = \begin{cases} 1/2, \text{for } |t| < 1 \\ 0, \text{otherwise} \end{cases}$; (C) $f(t) = \begin{cases} 1, \text{for } |t| < 1 \\ 0, \text{otherwise} \end{cases}$;

(D) $f(t) = \begin{cases} 1, \text{for } |t| < 1/2 \\ 0, \text{otherwise} \end{cases}$; (E) None of the above.

10. (5%) You can expand the function defined by $f(x) = x^2 + 3, 0 < x < 3$ in a Fourier series, a cosine series or a sine series. Please choose the **correct answers**
(A) $f(6)=3$ for sine series; (B) $f(3)=12$ for cosine series; (C) $f(0)=3$ for Fourier series;
(D) $f(-1)=4$ for Fourier series; (D) $f(-3)=12$ for cosine series.

11. (5%) Solve the initial value problem:

$$y'+y = f(t), y(0) = 5, \text{ where } f(t) = \begin{cases} 0, \text{for } 0 \le t < \pi \\ 3\cos(t), \text{ for } t \ge \pi \end{cases}.$$

Please choose the **correct answers**

(A) $y(t) = 5e^{-t}, 0 \le t < \pi$; (B) $y(t) = 5e^{-t} + \frac{3}{2}e^{-(t+\pi)}, 0 \le t < \pi$;

(C) $y(t) = 5e^{-t} + \frac{3}{2}e^{-(t+\pi)} + \frac{3}{2}\sin(t+\pi) + \frac{3}{2}\cos(t+\pi), t \ge \pi$;

(D) $y(t) = 5e^{-t} + \frac{3}{2}e^{-(t-\pi)} + \frac{3}{2}\sin(t) + \frac{3}{2}\cos(t), t \ge \pi$;

(E) $y(t) = 5e^{-t} + \frac{3}{2}e^{-(t-\pi)} + \frac{3}{2}\sin(t-\pi) + \frac{3}{2}\cos(t-\pi), t \ge \pi$

12. (15%) Please solve the differential equation $(x^2 + 2xy - y^2)dx + (y^2 + 2xy - x^2)dy$
(a) (3%) please verify that the differential equation is exact or not.
(b) (5%) please show that the integrating factor $\mu(x,y) = (x+y)^{-2}$
(c) (7%) please solve the differential equation.

13. (10%) As illustrated in the Figure 13, light rays strike a plane curve C in such a manner that all rays L parallel to the x-axis are reflected to a single point O.
(a) (5%) Assume that the angle of incidence is equal to the angle of reflection, determine a differential equation that describes the shape of the curve C.
[Hint: Please show that we can write $\phi=2\theta$ firstly.]

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(b) (5%) Please solve the differential equation to get the function describing the curve C.

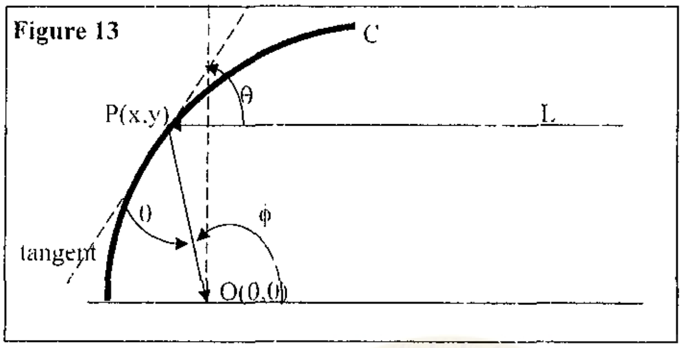

Figure 13

14. (10%) In the paraxial approximation, the light ray trajectory is almost parallel to the z-axis. The light ray equation can be expressed as

$$\frac{d}{dz}\left(n\frac{dy}{dz}\right)=\frac{dn}{dy},\ \frac{d}{dz}\left(n\frac{dx}{dz}\right)=\frac{dn}{dx}$$ where $n = n(x,y,z)$ is the refractive index.

(a) (2%) In a homogeneous medium where n is independent of x, y, z, please show that the light ray trajectory is a straight line.

(b) (8%) Let a light ray be incident into a slab graded index medium, in which $n = n_0(1-\alpha^2 y^2)$ with $\alpha^2 y^2 << 1$, at position $y = y_0$ and with an incidence angle $\frac{dy}{dz} \cong \theta_0$. Please show that with appropriate approximation the light ray trajectory is a periodic function as in Figure 14 and find the period.

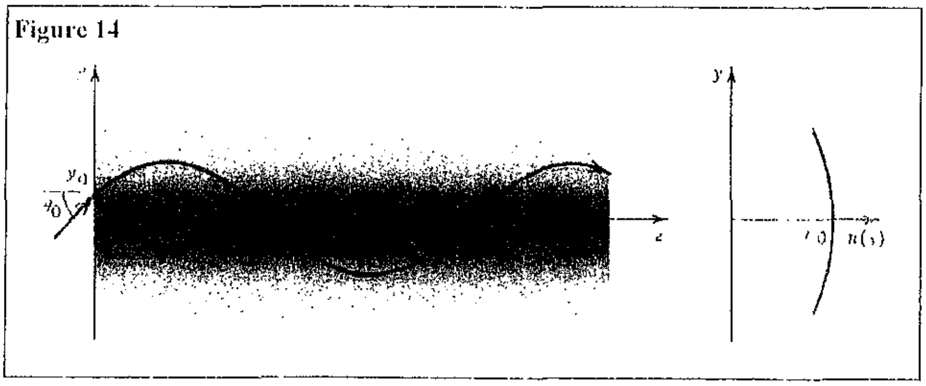

Figure 14

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