國立臺灣大學98學年度碩士班招生考試試題

GEOMETRY

Total points: 100

1 (25 pts). Let S be the torus in R^3 obtained by rotating about the z-axis the circle C_0: $(y-r)^2+z^2=a^2$ $(r>a>0)$ in yz-plane, and the circle C be the trajectory of the point $P=(0,r,a)$ of C_0 under the rotation. i) Compute the absolute value of the geodesic curvature of C at any $Q \in C$. ii) Given the vector $v_0=(0,1,0) \in T_P(S)$, let v_f be the vector back at P after the parallel transport of v_0, in S, by going around C once. Find explicitly, with proof, the vector v_f.

2 (30 pts). Write M_h for planes parallel to xy-plane, and M_v for planes containing z-axis. Let $S:\ z=x^2+y^2$ be the paraboloid of revolution, and $\gamma_h=M_h \cap S$, $\gamma_v=M_v \cap S$. i) Is γ_h a geodesic? Prove your answer. ii) Is γ_v a geodesic? Prove your answer. iii) Discuss the question whether there exists any closed geodesic in S.

3 (25 pts). Let S be a surface (orientable and connected), and γ_1, γ_2 be smooth curves in S. Assume that γ_1, γ_2 be simple closed geodesics (a simple closed curve means a closed curve with no further self-intersections), and $\gamma_1 \cap \gamma_2 = \emptyset$. Prove the following. i) If S is compact without boundary, then S cannot be of positive curvature everywhere. ii) If S is homeomorphic to a cylinder, then S cannot be of negative curvature everywhere.

4 (20 pts). Let $C_1=H_1 \cap S$, $C_2=H_2 \cap S$ be curves through a point $P \in S$, sharing the same tangent line l at P, where S is a surface, and H_1, H_2 are planes intersecting each other along l and making an angle θ with each other. Suppose that H_1 contains the normal N to S at P (so C_1 is a normal section of S at P), and that C_1 is a circle of radius r. Find, with proof, the absolute value of geodesic curvature of C_2 at P, in terms of r and θ.

試題隨卷繳回