Instructions:

- Write in Chinese or English only.
- Make sure all your answers are legible, unquestionably labeled, and clearly explained (with equations if possible).
- A standard normal probability table is attached on the last page.

1. **(16 points)** A retail chain has set each of its stores a target of earning \$2 million in revenue during a month. The 100 stores are in locations with similar demographics. The head office believes that each store has a probability of 0.4 of reaching the revenue target, and that different stores' performances are independent of one another. The managers of a store that reaches the revenue target receive \$80,000 in bonuses.

   (a) **(8 points)** Calculate the expected value and standard deviation of the number of stores that will reach the revenue target.

   (b) **(8 points)** Calculate the probability that the chain pays out more than \$3 million in bonuses.

2. **(18 points)** The admissions office at a university is determining how much weight to put on an applicant's high school GPA in making admissions decisions. It has collected the high school GPAs ($GPAHS$) and the college first-year GPAs ($GPAFY$) of a random sample of 75 students who just completed their first year at the university. Table 1 gives the summary statistics.
   Assume that the sample satisfies the assumptions of the random sampling regression model. The estimation results for the linear regression $\widehat{GPAFY}_i = \beta_0 + \beta_1 GPAHS_i$ are summarized in Table 2.

Table 1: Summary Statistics

Variable	Obs	Mean	S.D.	Min	Max
$GPAFY$	75	2.97	0.51	1.77	4.00
$GPAHS$	75	3.44	0.32	2.62	4.00

Table 2: Regression Estimates

Variable	Estimate	Std. Error
$GPAHS$	0.3109	0.1491
Constant	1.9046	0.5106

$n = 75$. The dependent variable is $GPAFY$. Heteroskedasticity-robust standard errors are presented.

   (a) **(6 points)** Do the data provide convincing evidence that a student's high-school GPA is associated with his or her college first-year GPA? State the hypotheses and conduct the test at a 5% significance level.

   (b) **(6 points)** A student improves her high-school GPA by 1.2. Construct a 90% confidence interval for change in her college first-year GPA.

   (c) **(6 points)** The office estimates a different model $\widehat{GPAHS}_i = \gamma_0 + \gamma_1 GPAFY_i$ using the data. Calculate the least-squares estimate of the slope coefficient $\hat{\gamma}_1$.

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3. **(16 points)** A chain of convenience stores wants the stores' stocks of bottled water to average 50 boxes at any given time, and plans to change its inventory policy if there is strong evidence that such an average is not being maintained.

   (a) **(4 points)** State appropriate null and alternative hypotheses in symbols and in words for this scenario.

   (b) **(6 points)** Suppose that the chain obtains the current bottled water inventories from 82 randomly selected stores. The sample mean is 52.5 boxes, and sample standard deviation is 9.8 boxes. Find the p-value and evaluate whether to reject the null hypothesis using a significance level of 0.05.

   (c) **(6 points)** If the true mean inventory is 54 boxes, what is the probability that the test from part (b) results in a Type II error?

4. **(30 points)** Please answer the following questions with either "**True**" or "**False**," then briefly justify your answers. Answers without justifications will receive *no* points.

   $\{y_i\}_{i=1}^n$ are random variables such that for $i = 1, 2, \ldots, n$, $\mathbb{E}(y_i) = b_0 + b_1 x_{i1} + b_2 x_{i2} + b_3 x_{i3}$ for some $\mathbf{b} = [b_0, b_1, b_2, b_3]^\top$, where $\{\mathbf{x}_i\}_{i=1}^n = \{[1, x_{i1}, x_{i2}, x_{i3}]^\top\}_{i=1}^n$ are non-random and *not* multicollinear, $\text{var}(y_i) = \sigma_0^2$, and $\text{corr}(y_i, y_j) = \rho$ for $j \neq i$.

   (a) **(6 points)** Suppose that $\rho \neq 0$. Then $\hat{\mathbf{b}}_{OLS} = [\hat{b}_0, \hat{b}_1, \hat{b}_2, \hat{b}_3]^\top$, the ordinary least squares (OLS) estimator, is a *biased* estimator for $\mathbf{b}$.

   (b) **(6 points)** If we have a sample size $n = 500$, then $\hat{\mathbf{b}}_{OLS} = [\hat{b}_0, \hat{b}_1, \hat{b}_2, \hat{b}_3]^\top$, the OLS estimator for $\mathbf{b}$, is *normally-distributed* in general.

   (c) **(6 points)** In the beginning we said that $\{\mathbf{x}_i\}_{i=1}^n = \{[1, x_{i1}, x_{i2}, x_{i3}]^\top\}_{i=1}^n$ are *not* multicollinear. It is important because weird things happen when the independent variables are multicollinear. For example, if $x_{i1} = 2x_{i2}$, then $\hat{b}_1 = 0.5\hat{b}_2$.

   (d) **(6 points)** Consider the *Eicker-White* variance-covariance matrix estimator

   $$\hat{D} = \left(\frac{1}{n}\sum_{i=1}^n \mathbf{x}_i\mathbf{x}_i^\top\right)^{-1} \left(\frac{1}{n}\sum_{i=1}^n \hat{\epsilon}_i^2 \mathbf{x}_i\mathbf{x}_i^\top\right) \left(\frac{1}{n}\sum_{i=1}^n \mathbf{x}_i\mathbf{x}_i^\top\right)^{-1},$$

   where $\hat{\epsilon}_i = y_i - \mathbf{x}_i^\top \hat{\mathbf{b}}_{OLS}$ is the residual. Suppose that $\rho = 0$, *i.e.*, $\{y_i\}_{i=1}^n$ are homoskedastic. Then $\hat{D}$ is an *inconsistent* estimator for the variance-covariance matrix of $\hat{\mathbf{b}}_{OLS}$.

   (e) **(6 points)** Suppose that $\{x_{i1}, x_{i2}, x_{i3}\}_{i=1}^n$ are random variables. Consider two models.

   $$y_i = c_0 + c_1 x_{i1} + u_i, \quad \text{(Model 1)}$$
   $$y_i = d_0 + d_1 x_{i1} + d_2 x_{i2} + v_i. \quad \text{(Model 2)}$$

   We estimate both models by the OLS estimator. Suppose $\hat{c}_1$, the OLS estimate for c_1 in Model 1, is numerically equivalent to $\hat{d}_1$, the OLS estimate for d_1 in Model 2, then we can conclude that $\{x_{i1}\}_{i=1}^n$ and $\{x_{i2}\}_{i=1}^n$ are independent.

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5. **(8 points)** Consider the following dynamic panel model. For $i = 1, 2, \ldots, n$ and $t = 1, 2, \ldots, T$,

$$y_{i,t} = \alpha_i + \beta y_{i,t-1} + \epsilon_{i,t},$$

where $|\beta| < 1$, and $\{\epsilon_{i,t}\}$ are independent and identically distributed (i.i.d.) with a finite second moment. To eliminate the heterogeneous intercepts $\{\alpha_i\}$, we take the first difference:

$$\Delta y_{i,t} = \beta \Delta y_{i,t-1} + \Delta \epsilon_{i,t},$$

where $\Delta y_{i,t} = y_{i,t} - y_{i,t-1}$. Let

$$\hat{\beta}_{FD} = \frac{\sum_{i=1}^{n} \sum_{t=2}^{T} (\Delta y_{i,t-1})(\Delta y_{i,t})}{\sum_{i=1}^{n} \sum_{t=2}^{T} (\Delta y_{i,t-1})^2}.$$

Suppose that $n \to \infty$ while T is fixed. Is $\hat{\beta}_{FD}$ a consistent estimator for β? Justify your answer. Answers without justifications will receive *no* points.

6. **(12 points)** Suppose that $\{y_i\}_{i=1}^{n}$ are generated according to

$$y_i = \beta_0 x_i + \epsilon_i,$$

where $\{[x_i, \epsilon_i]^\top\}_{i=1}^{n}$ are independent and identically distributed (i.i.d.) random vectors such that $\mathbb{E}(x_i) = \mu_x$, $\mathbb{E}(\epsilon_i) = 0$, $\text{var}(x_i) = \sigma_x^2$, $\text{var}(\epsilon_i) = \sigma_\epsilon^2$, and $\mathbb{E}(x_i \epsilon_i) = 0$.

Unfortunately, $\{x_i\}_{i=1}^{n}$ cannot be observed directly. Instead, we observe $\{w_i\}_{i=1}^{n}$ and $\{z_i\}_{i=1}^{n}$ with

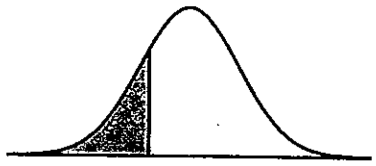
$$w_i = x_i + u_i, \text{ and } z_i = x_i + v_i,$$

where $\{u_i\}_{i=1}^{n}$ and $\{v_i\}_{i=1}^{n}$ are i.i.d. random variables such that $\mathbb{E}(u_i) = \mathbb{E}(u_i x_i) = \mathbb{E}(u_i \epsilon_i) = 0$, $\text{var}(u_i) = \sigma_u^2$, $\mathbb{E}(v_i) = \mathbb{E}(v_i x_i) = \mathbb{E}(v_i \epsilon_i) = 0$, $\text{var}(v_i) = \sigma_v^2$, and $\mathbb{E}(u_i v_i) = 0$.

(a) **(6 points)** Regress y_i on w_i without intercept and obtain the ordinary least squares (OLS) estimate $\tilde{\beta}$. Determine the probability limit of $\tilde{\beta}$.

(b) **(6 points)** How would you estimate β_0 consistently? Write down your estimator and briefly justify your answer. Answers without justifications will receive *no* points.

Standard normal probability table



negative Z

Second decimal place of Z										
0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	Z
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	−3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	−3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	−3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	−3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	−3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	−2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	−2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	−2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	−2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	−2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	−2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	−2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	−2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	−2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	−2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	−1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	−1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	−1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	−1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	−1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	−1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	−1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	−1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	−1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	−1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	−0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	−0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	−0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	−0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	−0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	−0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	−0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	−0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	−0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	−0.0

*For $Z \leq -3.50$, the probability is less than or equal to 0.0002.

試題隨卷繳回