Multiple-Choice (單一選擇題) ※ 注意：請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。
5 points for each question (一題五分)

1. Consider a diatomic molecule rotating about its center with eigenfunctions characterized by a φ dependence expressed as $\psi(\varphi) = Ae^{im\varphi}$. In this expression, m represents a quantum number and A is a normalization constant. Select the value of A that would correctly normalize this eigenfunction.

   (A) $(2\pi)^{-1/2}$ (B) 2π (C) $(2\pi)^2$ (D) $(2\pi)^{1/2}$ (E) $1/(2\pi)$

2. A Gaussian wave function $\psi(x)=A\exp[-bx^2/2]$, where A and b are real constants, is a normalized eigenfunction of the Schrödinger equation for a particle of mass m and energy E in a one-dimensional potential $V(x)$ such that $V(x)=0$ at $x=0$. Which of the following is correct?

   (A) $V=\hbar^2b^4/(2m)$ (B) $V=\hbar^2b^4x^2/(2m)$ (C) $V=\hbar^2b^6x^4/(2m)$ (D) $E=\hbar^2b^2(1-b^2x^2)$ (E) $E=\hbar^2b^6/(2m)$

3. Identify the option that does NOT align with the selection rule governing the emission of photons through electric dipole transitions in excited atomic states.

   (A) Δn may have any negative integral value (B) $\Delta\ell=\pm1$ (C) $\Delta m_\ell=0, \pm1$ (D) $\Delta s=\pm1$ (E) $\Delta j=\pm1$

4. The energy of the ground state of positronium is most closely equal to

   (A) −27.2 eV (B) −13.6 eV (C) −6.8 eV (D) −3.4 eV (E) 13.6 eV

5. Consider two ions, labeled 1 and 2, positioned at a constant distance from each other. Each ion has spin angular momentum operators $\mathbf{S_1}$ and $\mathbf{S_2}$. Their interaction is described by the Hamiltonian $H = -J\mathbf{S_1}\cdot\mathbf{S_2}$, where J is a positive constant. Given that the values of $\mathbf{S_1}^2$ and $\mathbf{S_2}^2$ are fixed at $S_1(S_1+1)$ and $S_2(S_2+1)$ respectively, what is the energy of the ground state of this system?

   (A) 0 (B) $-JS_1S_2$ (C) $-J[S_1(S_1+1)-S_2(S_2+1)]$ (D) $-(J/2)[(S_1+S_2)(S_1+S_2+1)-S_1(S_1+1)-S_2(S_2+1)]$
   (E) $-(J/2)[S_1(S_1+1)+S_2(S_2+1)]/[(S_1+S_2)(S_1+S_2+1)]$

6. In the hydrogen spectrum, what is the ratio of the longest wavelength in the Lyman series (with a final quantum number $n_f=1$) to the longest wavelength in the Balmer series (where $n_f=2$)?

   (A) 5/27 (B) 1/3 (C) 4/9 (D) 3/2 (E) 3

7. When solving the Schrödinger equation for a particle confined in a one-dimensional, infinitely deep potential well, the solutions are characterized by the quantum number n. For which values of n does the probability density become zero at the center of the well?

   (A) The ground state (n=1) only (B) States of even n (n=2, 4, ...) (C) States of odd n (n=1, 3, ...) (D) All states (n=1, 2, 3, ...)
   (E) All states except the ground state

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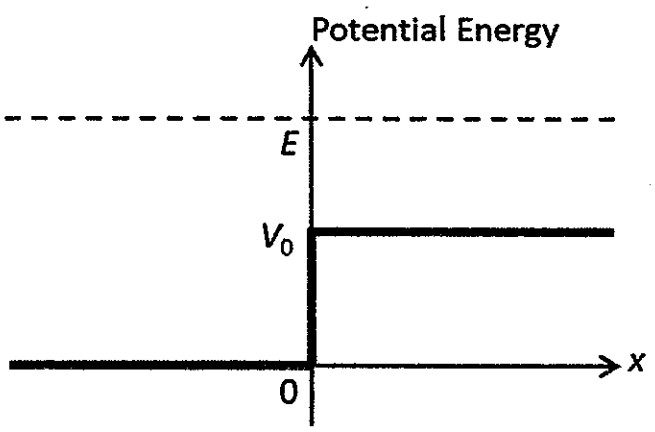
8. Consider an atom with completely filled n=1 and n=2 energy levels. What is the total number of electrons present in this atom?

(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

9. An electron of mass m with total energy E in the region x<0 is moving in the +x-direction. It encounters a step potential at x=0 as shown in the figure below.



The wave function for $x\leq 0$ is given by $\Psi=A\exp[ik_1x]+B\exp[-ik_1x]$, where $k_1=(2mE/\hbar^2)^{1/2}$, and A and B are coefficients.
The wave function for $x>0$ is given by $\Psi=C\exp[ik_2x]$, where $k_2=[2m(E-V_0)/\hbar^2]^{1/2}$, and C is a coefficient.
Which of the following gives the reflection coefficient R for the system?

(A) R=1 (B) R=0 (C) $R=k_1/k_2$ (D) $R=[(k_1-k_2)/(k_1+k_2)]^2$ (E) $R=4k_1k_2/(k_1-k_2)^2$

10. A spin-1/2 particle is in a state described by the spinor

$$\mu=A\begin{pmatrix}1+i\\2\end{pmatrix}$$

where A is a normalization constant. The probability of finding the particle with spin projection $S_z=-\hbar/2$ is

(A) 1/4 (B) 1/3 (C) 1/2 (D) 2/3 (E) 1

11. According to the Bohr model of the hydrogen atom, what expression represents the linear momentum of the electron at a radius r_n, where n is the principal quantum number?

(A) $n^2\hbar$ (B) $nr_n\hbar$ (C) $n\hbar/r_n$ (D) $n^2r_n\hbar$ (E) $n^2\hbar/r_n$

12. The state $\psi=\frac{1}{\sqrt{6}}\psi_{-1}+\frac{1}{\sqrt{2}}\psi_1+\frac{1}{\sqrt{3}}\psi_4$ is a linear combination of three orthonormal eigenstates of the operator $\hat{O}$ corresponding to eigenvalues −1, 1, and 4. What is the expectation value of $\hat{O}$ for this state?

(A) 2/3 (B) $(7/6)^{1/2}$ (C) 1 (D) 5/3 (E) $11^{1/2}$

13. Consider a free particle that has a kinetic energy E and a de Broglie wavelength λ. When this particle enters a region where its potential energy is V, what becomes its new de Broglie wavelength?

(A) $\lambda(1+E/V)$ (B) $\lambda(1-V/E)$ (C) $\lambda(1-E/V)^{-1}$ (D) $\lambda(1+V/E)^{1/2}$ (E) $\lambda(1-V/E)^{-1/2}$

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14. The components of the orbital angular momentum operator, denoted as $L=(L_x, L_y, L_z)$, adhere to the following commutation relations
$[L_x, L_y]=i\hbar L_z$,
$[L_y, L_z]=i\hbar L_x$,
$[L_z, L_x]=i\hbar L_y$,
What is the value of the commutator $[L_xL_y, L_z]$?

(A) $i\hbar(L_x^2+L_z^2)$ (B) $i\hbar(L_x^2+L_y^2)$ (C) $-i\hbar(L_x^2+L_y^2)$ (D) $i\hbar(L_x^2-L_y^2)$ (E) $-i\hbar(L_x^2-L_y^2)$

15. In experiments conducted deep underground, which two types of cosmic rays are most commonly observed to reach the experimental apparatus

(A) alpha particles and neutrons (B) protons and electrons (C) iron nuclei and carbon nuclei
(D) muons and neutrinos (E) positrons and carbon nuclei

16. A rocket, whose rest length is 60 m, is moving directly away from the Earth. The rocket is fitted with mirrors at each end. A light signal, sent from the Earth, is reflected back from the two mirrors. The second light signal is received 1.74 μs after the first light signal was received. What is the speed of the rocket relative to the Earth? (c is the light speed.)

(A) 0.007 c (B) 0.115 c (C) 0.230 c (D) 0.973 c (E) 0.993 c

17. In the inertial system S, an event is observed to take place at point A on the x-axis and 1×10^{-6} s later another event takes place at point B, 900 m further down. Find the speed of another system S' moving in the positive x direction with respect to S in which these two events appear simultaneous. (Use light speed $c = 3.00\times10^8$ m)

(A) 0.099 c (B) 0.300 c (C) 0.316 c (D) 0.562 c (E) 0.949 c

18. The decay time of a muon, in its rest frame of reference, is 2×10^{-6} s. For a muon formed 6.000 km above the ground of the Earth and heading straight to the ground, which one below is the closest to and greater than the minimal speed in which it has to travel so that it may reach the ground as a muon? (c is the light speed.)

(A) 0.9801 c (B) 0.9899 c (C) 0.9901 c (D) 0.9909 c (E) 0.9991 c

19. A rocket is moving directly away from the Earth in a speed $v=24c/25$. Information it gathers is sent to the Space Center on Earth with a radio-wave signal emitter on it. The emitter frequency as determined on the rocket is 100 MHz. What is the frequency of the radio-wave signal received on Earth? (Use light speed $c=3.00\times10^8$ m)

(A) 14.3 MHz (B) 37.8 MHz (C) 96.0 MHz (D) 102 MHz (E) 265 MHz

20. A particle of rest mass m_1 and 3-momentum $\mathbf{P}_1$ collides inelastically with a particle of mass m_2 at rest in the laboratory. The two particles stick together, forming a particle of mass m_3. Find the velocity of the resulting particle relative to the laboratory-frame. (c is the light speed.)

(A) $c\mathbf{P}_1/(m_1^2c^2+p_1^2+m_2^2c^2)^{1/2}$ (B) $c\mathbf{P}_1/(m_1^2c^2+p_1^2+m_2^2c^2+m_3^2c^2)^{1/2}$ (C) $c\mathbf{P}_1/(m_1^2c^2+p_1^2+m_2^2c^2-m_3^2c^2)^{1/2}$
(D) $c\mathbf{P}_1/[(m_1^2c^2+p_1^2)^{1/2}+m_2c]$ (E) $c\mathbf{P}_1/[(m_1^2c^2+p_1^2)^{1/2}+m_2c+m_3c]$

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