1.Find the general solution to the following O.D.E. **(20%)**:

$$\text{(i) } y'' - 3y' + 2y - x - e^{-x} = 0$$

$$\text{(ii) } y' = -\left(\frac{y}{x}\right) + x - 5$$

2.

$$m\frac{d^2y}{dx^2} + kz = F_0 \cos\omega t$$

,where F_0 is the forcing coefficient and ω is the forcing frequency.
What is the motion of the mass with time, $z(t)$? **(15%)**

3. Given are the expressions for the collision frequency in the continuum regime ($\beta_c(u,v)$) and the free molecular regime ($\beta_{FM}(u,v)$):
For the continuum regime:

$$\beta_c(u,v) = \frac{2k_BT}{3\mu}\left(\frac{1}{u^{1/3}} + \frac{1}{v^{1/3}}\right)\left(u^{1/3} + v^{1/3}\right)$$

For the free molecular regime:

$$\beta_{FM}(u,v) = \left(\frac{3}{4\pi}\right)^{1/6}\left(\frac{6k_BT}{\rho}\right)^{1/2}\left(\frac{1}{u} + \frac{1}{v}\right)^{1/2}\left(u^{1/3} + v^{1/3}\right)^2$$

Here, u and v is the velocity in x and y direction, respectively.
(i) Determine the condition (i.e. express "u" in terms of "v") for the maxima/minima of the collision frequency functions ($\beta_c(u,v)$ and $\beta_{FM}(u,v)$). Indicate whether this condition corresponds to a maximum or a minimum. **(10%)**
(ii) Solve for "u" in terms of "v" by equating β_c and β_{FM}. **(5%)**

4. (i) Find derivative of $y = 2^{\sqrt{x}}$. **(7%)**

(ii) Find a power series representation of the function given below and find the interval of convergence of the series. **(8%)**

$$f(x) = \frac{2x}{1+x}$$

5. Solve $(2x+y)\frac{\mathrm{dy}}{\mathrm{dx}} = x + 2y$. **(10%)**

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6.(i) Show that the general solution of $y'' + Ay' + By = 0$, can be written in the form of hyperbolic functions:

$$y(x) = [c_1 \cosh(\beta x) + c_2 \sinh(\beta x)]\, e^{\alpha x}$$

where α and β will be the function of A and B. For appropriate choice of α and β, assuming that $A^2 - 4B > 0$. **(10%)**

(ii) Solve $9y'' + 3y' - 2y = 0$ in terms of hyperbolic functions. **(5%)**

7. Applying the Laplace transform, solve the following equation: **(10%)**

$$y'' - 3y' + 10y = 1;\ \ y(0) = -1,\ y'(0) = 2$$

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