1. [20 points] Consider the electron radiation process

$$e(p) \to e(p') + \gamma(q)$$

where p, p' and q are their 4-momenta and all these particles are on shell.

(a) [10] Use appropriate equations to explain why this process cannot happen *in vacuum*.

(b) [10] Use appropriate equations to explain why this process can possibly happen *in a detector* (*e.g.*, the Cerenkov radiation). Specify under what condition does that happen.

2. [30 points] A box containing a particle is divided into a left compartment and a right compartment by a thin partition. If the particle is known to be on the left (right) side with certainty, the state is represented by the position eigenket $|L\rangle$ ($|R\rangle$). A general state vector can be expressed as

$$|\alpha\rangle = |L\rangle \langle L|\alpha\rangle + |R\rangle \langle R|\alpha\rangle \ .$$

Suppose it is possible for the particle to tunnel through the partition or remain in the original partition. as characterized by the Hamiltonian

$$\mathcal{H} = \Delta \left(|L\rangle \langle L| + |L\rangle \langle R| + |R\rangle \langle L| + |R\rangle \langle R|\right) \qquad \text{with } \Delta \in \mathbb{R} \ .$$

(a) [10] Find the normalized energy eigenstates and the corresponding energy eigenvalues. For the normalized eigenstates, write them in the form of $N \begin{pmatrix} 1 \\ a \end{pmatrix}$, where N and a are to be derived.

(b) [10] What is the matrix that diagonalizes $\mathcal{H}$, with the elements of the diagonalized $\mathcal{H}$ listed in the ascending order?

(c) [10] At time $t = 0$, the system is in the $|R\rangle$ state and the energy is measured. What are the possible outcomes of the energy measurement and with what probabilities?

3. [10 points] According to Planck's postulate, the energy of the pendulum executing simple harmonic oscillations is quantized. Here, one pendulum consisting of a 0.01 kg mass is suspended from a 1 m long string. The pendulum in its extreme positions makes an angle of 0.1 rad from the vertical and its energy will decrease due to friction. What is the sensitivity required for the experiment equipment to detect the discontinuous changes in energy (Please estimate the order of magnitude in energy)?

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4. [10 points] Suppose the speed of a bullet ($m = 0.05$ kg) and an electron are measured to be the same, namely 300 m/sec, with an uncertainty of 0.01%. The positions of the bullet and electron are measured simultaneously with their speed in the same experiments. What accuracy could we locate

   (a) [5] the position of the bullet?

   (b) [5] the position of the electron?

5. [10 points] The hydrogen atom has magnetic properties.

   (a) [4] Calculate the orbital magnetic dipole moment μ_ℓ in terms of the orbital angular momentum L

   (b) [3] What is the z-component of quantization of magnetic moment $\mu_{\ell z}$?

   (c) [3] What is the z-component of intrinsic magnetic dipole moment of an electron μ_{sz}? (μ_B: Bohr magneton; m_ℓ: magnetic orbital projection quantum number; m_s: spin projection quantum number)

6. [10 points] Calculate the ground state of Cr^{2+} $(Cr : 1s^2 2s^2 2p^6 3s^2 3p^6 3d^5 4s^1)$ by Hunds' rules:

   (a) [3] What is the maximum value of the spin angular momentum S?

   (b) [3] What is the maximum value of the orbital angular momentum L?

   (c) [4] What is the value of the total angular momentum J?

7. [10 points] X-ray crystallography is an important tool for determining the atomic and molecular structure of a crystal.

   (a) [3] W. L. Bragg presented a simple explanation of the diffracted beams from a crystal. What is Bragg's law?

   (b) [3] Figure 1 shows X-ray diffraction pattern of MgO. MgO is the simple cubic structure and lattice constant is 4.211$\mathring{A}$. Please calculate the parallel lattice plane space, d_{hkl} , when hkl = 422.

   (c) [4] Please find the cross angle of two planes between (222) and (420).

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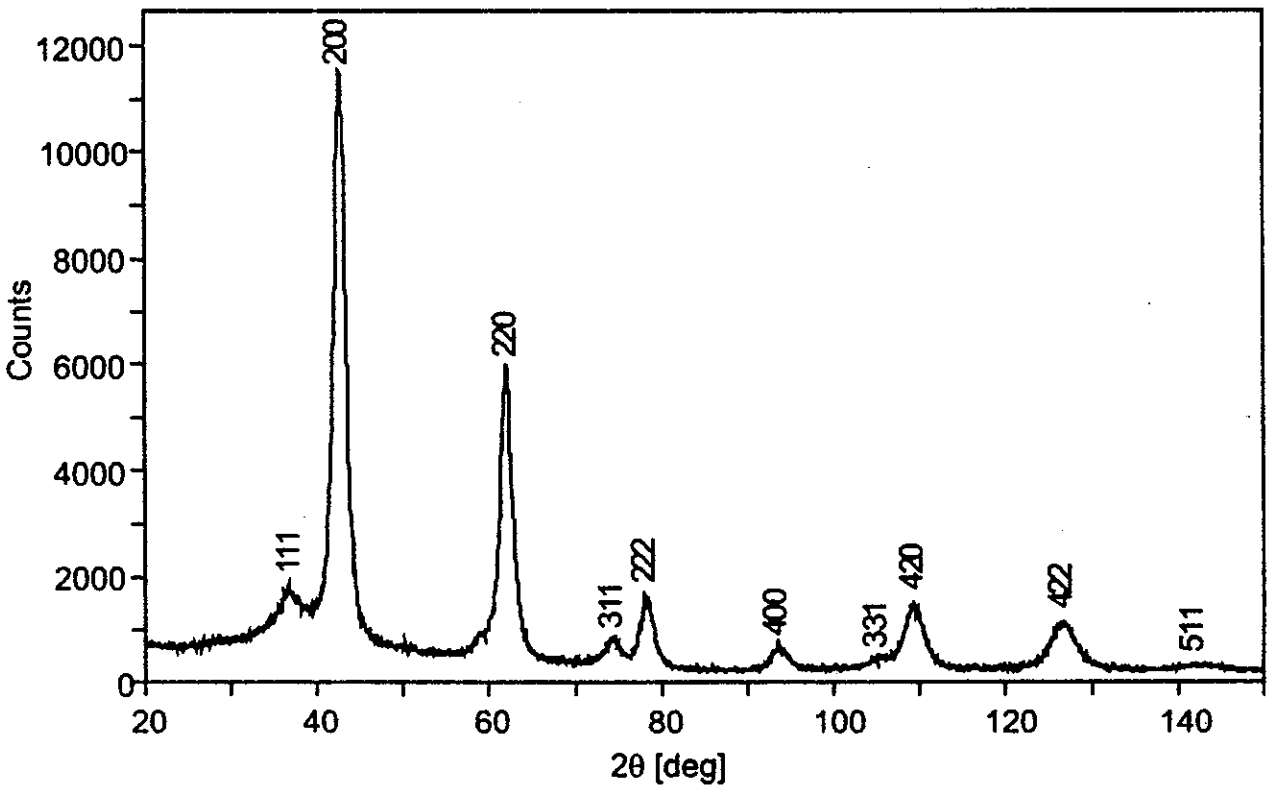

. 1: X-ray diffraction of powder MgO (G. Dercz et al., Materials Science-Poland, 27, 1, 2009)

:: The following physical constants and conversion factors may be useful in numerical calculations.

Quantity	Symbol	Value
speed of light	c	3×10^8 m/s
electron charge	e	1.6×10^{-19} C
Planck constant	h	6.634×10^{-34} J s
vacuum permittivity	ξ_0	$8.85 \times 10^{-12} C^2/\mathrm{N} \cdot m^2$
Bohr magneton	$\mu_B = \frac{e\hbar}{2m_e}$	$9.27 \times 10^{-24} \frac{\mathrm{J}}{\mathrm{T}}$
electron mass	m_e	9.11×10^{-31} kg $= 0.511$ MeV/c^2
proton mass	m_p	1.673×10^{-27} kg $= 938.27$ MeV/c^2

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