國立臺灣大學 111 學年度碩士班招生考試試題

Instructions:

- Each answer need to be clearly labeled on the answer sheet.
- Problems 1 to 16 are worth 5 points each. Only the answer will be graded.
- Problems 17 and 18 are worth 10 points each. Show all work and explanations.
- Use of any device with computer algebra system during the exam is not allowed.

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1. Compute the limit $\lim_{x\to\infty}\left(\cos\left(\frac{2}{x}\right)+\frac{2}{x^2}\right)^{x^4} =$ __(1)__.
2. Compute the limit $\lim_{x\to 0^+}\frac{\tan^{-1}(2x)-\tan(2x)}{\sin(x)-x} =$ __(2)__.
3. The graph of $f(x)=x^2e^{-2x}$ has two inflection points. Find one: __(3)__.
4. If $\int_3^{2x+1} e^{f(t)+1}\,dt=\ln x$, then $f(4)=$ __(4)__.
5. Calculate the integral $\int_0^3 \frac{x^3}{x^2+9}\,dx =$ __(5)__.
6. Calculate the integral $\int_0^{\ln 5}\frac{dx}{e^x+1} =$ __(6)__.
7. Calculate the integral $\int_0^3 \sqrt{4x-x^2}\,dx =$ __(7)__.
8. Suppose that $\frac{dy}{dx}+2xy=3x$ with $y(0)=5$, then $y(2)=$ __(8)__.
9. Let R be the region below the curve $y=\sin\left(x^2\right)$ when $0\le x\le\sqrt{\pi}$ and V be the volume of the solid obtained by rotating R about the y-axis. $V=$ __(9)__.
10. The interval of convergence of $\sum_{n=0}^{\infty}\frac{(3-5x)^n}{2^n\sqrt{n+1}}$ is __(10)__.
11. Find the sum $\sum_{n=5}^{\infty}\frac{1}{n}\left(\frac{2}{3}\right)^n =$ __(11)__.
12. Find the sum $\sum_{n=0}^{\infty}(-1)^n\frac{1+3^n}{n!} =$ __(12)__.
13. Let $\vec{r}(t)=(\cos(e^t),\sin(e^t),3e^t)$, $-2\le t\le 2$. The length of the curve is __(13)__.

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14. The maximum value of the function $f(x,y) = 3x^2 + 4y^2$ inside the region $x^2 + y^2 \le 9$ is (14).

15. $\int_0^3 \int_0^2 \int_{y/2}^1 x^2 y \cos\left(z^3 - 1\right) dz\, dy\, dx =$ (15).

16. The work done by the force field $\vec{F}(x,y) = (x^2y)\vec{i} - (xy^2)\vec{j}$ in moving a particle clockwise around the circle $x^2 + y^2 = 4$ once is (16).

17. A rectangular box without a lid is to be made from 6 m^2 of cardboard. Find the maximum volume of such a box.

18. Evaluate the outward flux of the vector field $\vec{F}(x,y,z) = (z,y,x)$ over the unit sphere $x^2 + y^2 + z^2 = 1$ with two methods: (a) Divergence Theorem and (b) Surface Integral.

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