1. **(20 points)** A manufacturer produces two products subjected to two types of resources and sets the selling prices at \$6 and \$10. The production quantity decision for these products (x_1 and x_2) of maximizing the revenue can be formulated as the following linear programming (LP) model:

$$\max z = 6x_1 + 10x_2$$
subject to
$$x_1 + 2x_2 \leq 15$$
$$2x_1 + 3x_2 \leq 25$$
$$x_1, x_2 \geq 0$$

   (a) (10 points) Since the Consumer Price Index surges by most recently, the manufacturer considers to raise the prices for both products by λ ($\lambda \geq 0$), and thus the selling prices will become \$6 + λ and \$10 + λ. Find out how λ affects the objective value by using parametric programming.

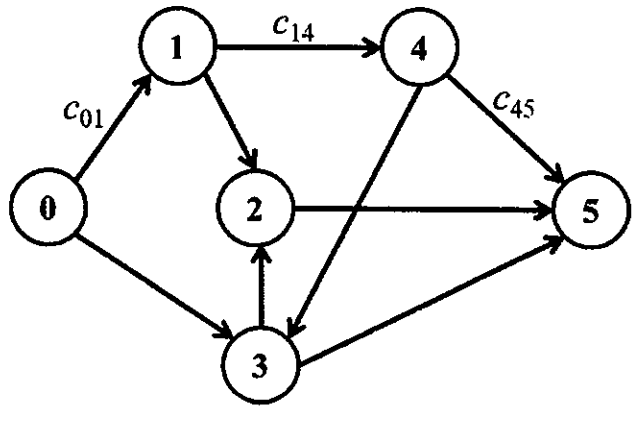
   (b) (10 points) Because of delay of the shipment, the availability of the resource used for the first constraint is affected ($x_1 + 2x_2 \leq 15 - b_1$). Determine how b_1 in the range of $0 \leq b_1 \leq 60$ affects the objective value while the selling prices remains the same as \$6 and \$10.

2. **(15 points)** A company develops 4 possible new products (say, products A, B, C, and D) and wants to decide which of these products will actually be produced and at what levels. If product i is manufactured, the corresponding setup cost, f_i will be incurred and the selling price will be set at p_i (i = A, B, C, and D). In addition, based on the marketing survey, some guidelines for the production plan are suggested:

   - At most three products can be produced.
   - If product A is produced, product B can't be produced.
   - If product C is produced, product D should also be produced.
   - Product A can be produced only if either product C or D is produced.

   Please formulate a mixed integer programming model to maximize the profit.

3. **(15 points)** There is a communication network consisting of 6 nodes and 9 directed arcs where c_{ij} denotes the arc cost linked nodes i and j as shown below.



   (a) (5 points) List all possible paths starting from node **0** to node **5**.

   (b) (10 points) Use the information obtained from (a) to formulate a programming model that disconnects the communication from node **0** to node **5** with the minimal cost.

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4. **(15 points)** The **IIE** produces two similar types of laptops, *Laptop A* and *Laptop B*. It costs $6q_1$ dollars to produce q_1 units of *Laptop A*, and it costs $3q_2^2$ dollars to produce q_2 units of *Laptop B*. Let $q = q_1 + q_2$, and q represent the total number of laptops produced. Since the two types of laptops are similar, all of the laptops will be sold for the same price and consumers will pay $1200 - 6q$ dollars for each unit of the laptops. If **IIE** wants to maximize the total profits, how many *Laptop A* and *Laptop B* should be produced?

5. **(15 points)** The **NTU farm** decided to keep m goats on-site at the end of the last year. And, at the end of each year, **NTU farm** will need to decide how many goats to sell or keep. The profit from selling a goat in the year i is p_i, and the goats kept at the end of a year i will double in number at the end of the next year. The **NTU farm** plans to sell all the goats at the end of the n^{th} year. The objective is to maximize the total profit over the n years.
   (a) (7 points) Develop a general dynamic programming model for this problem and clearly define all the variables and the recursive equation for your model.
   (b) (8 points) Solve the problem for the **NTU farm** when $m = 2$ goats, $n = 3$ years, $p_1 = \$50, p_2 = \$65$, and $p_3 = \$60$.

6. **(10 points)** The **NTUIIE** sells printers to NTU students. The inventory policy of the **NTUIIE** is to have at least 2 printers at the beginning of a week. The demand per week is estimated to be: 0 with probability 0.25, 1 with probability 0.3, 2 with probability 0.35, and 3 with probability 0.1. Unfilled demands will be backlogged and fulfilled in the next period. The inventory policy is to place an order of 4 printers whenever the inventory level drops below 2 printers at the end of a week. When an order is placed at the end of a week, the supplier will deliver the printers to **NTUIIE** at the beginning of the next week.
   (a) (5 points) Model the inventory level of **NTUIIE** as a Markov chain.
   (b) (5 points) Suppose that the week starts with 3 printers in stock. Find the probability that an order will be placed at the end of the next week.

7. **(10 points)** The **NTU Mart** is trying to decide how many cash registers to keep open. Suppose that the customer arrival process is a Poisson process with a rate of 0.6 customers per minute. Let the checkout time for a customer be exponentially distributed, and the average checkout time is 2 minutes per customer. It costs $200 per hour to operate a cash register, and the waiting cost of customers is assessed to be $2 per customer per minute.
   (a) (5 points) Model the number of customers in the cash register area as a queueing system.
   (b) (5 points) How many registers should the **NTU Mart** open to minimize the expected cost?

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