1. Solve the following ordinary differential equations:

(a) (10%) $x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+y=0$

(b) (10%) $\frac{d^2y}{dx^2}-y=e^x$

2. (10%) Solve the following integral equation:

$$y(t)+\int_0^t y(t-\tau)d\tau=1$$

3. For the following ordinary differential equation:

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(25x^2-1)y=0,\quad x\in[0,1],$$

(a) (10%) Find the general solution of it.

(b) (10%) If y(1) = 2, and y(x) is a finite physical quantity everywhere in the domain [0,1], then what is the particular solution of the above ordinary differential equation?
(Note: You can express the solutions to (a) and (b) in terms of special functions directly.)

4. (a) (10%) Find the Fourier Transform, $f(t)=4H(t-2)e^{-3t}\cos(t-2)$
(Note: H(t-2) is Step Function, also called Heaviside function)

(b) (10%) Find the inverse Fourier Transform, $F(\omega)=\frac{e^{(20-4\omega)i}}{3-(5-\omega)i}$

5. (15%) Solve the heat equation,

$$\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2}\quad 0<x<9,\ t>0$$
$$u(0,t)=T_1,\quad u(9,t)=T_2$$
$$u(x,0)=x^2$$

6. (15%) Solve the boundary value problem,

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0 \qquad (0<x<1,\ 0<y<4,\ 0<z<2)$$

If u=0 on the faces of cube x=0, x=1, z=0 and z=2, and u(x,0,z)=f(x) and u(x,4,z)=g(z)

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$$\int x\sin(ax)dx=\frac{\sin(ax)}{a^2}-\frac{x\cos(ax)}{a}\qquad \int x\cos(ax)dx=\frac{\cos(ax)}{a^2}+\frac{x\sin(ax)}{a}$$

$$\int x^2\sin(ax)dx=\frac{2x}{a^2}\sin(ax)+\left(\frac{2}{a^3}-\frac{x^2}{a}\right)\cos(ax)$$

$$\int x^2\cos(ax)dx=\frac{2x}{a^2}\cos(ax)+\left(\frac{x^2}{a}-\frac{2}{a^3}\right)\sin(ax)$$

試題隨卷繳回