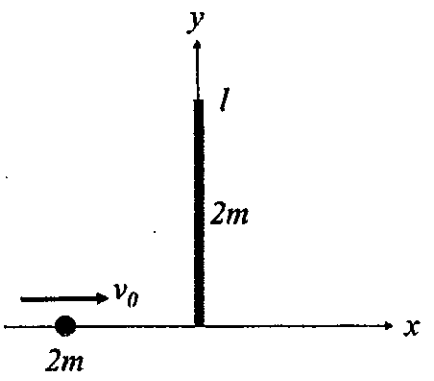
1. (25%) A uniform rod of mass $2m$ and length l rests along the y-axis on a horizontal table without friction. A particle of mass $2m$ is moving along the x-axis at a speed v_0. At $t = 0$ the particle strikes the end of the rod and sticks to it.

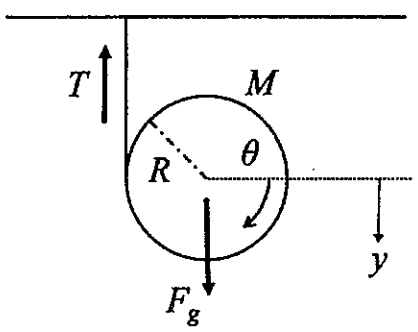


(a) What is the vector expression for the position of the center of mass of the system as a function of time, $R(t)$? (9%)

(b) Please find the angular velocity ω of the rod about its new center of mass immediately after the collision. (8%)

(c) What is the velocity v of the attached particle immediately after the collision? (8%)

2. (25%) A string (mass is negligible) attached to the bottom of a table is wrapped around a homogeneous cylinder of radius R and mass M. The cylinder is dropped from the test and rotates as the string unwinds at time t = 0.

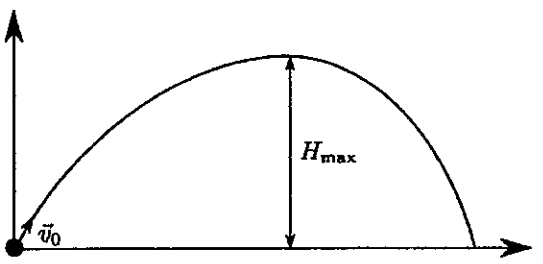


(a) Please find the tension T in the string. (8%)

(b) What is the linear acceleration A and angular acceleration α of the rotating cylinder? (9%)

(c) What is the velocity V and angular velocity ω of the cylinder's center? (8%)

3. (25%) A rock of mass $M = 1\text{kg}$ is tossed from the origin into the air at time $t = 0$. The initial velocity of the rock is $\vec{v}_0 = (4.9\text{m/s},\ 4.9\text{m/s})$. In addition to the force of gravity, the motion is subject to an air drag force, $\vec{f}_{drag} = -b\vec{v}$. Here, we assume that the gravitational acceleration $g = 9.8\text{m/s}^2$ and the drag constant $b = 2\text{kg/s}$. Please find the maximum height of this motion, $H_{\max}$.



4. (25%) A block of mass $M = 1\text{kg}$ is subject to a linear restoring force $f_{\text{spring}} = -kx$ and a periodic external force $f_{\text{ex}} = \frac{F_0}{\sqrt{2}}[\cos(\Omega t) + \sin(\Omega t)]$. Thus, the equation of motion can be written as $M\ddot{x} = -kx + \frac{F_0}{\sqrt{2}}[\cos(\Omega t) + \sin(\Omega t)]$. We know that $x(t = 0) = 0$ and $x\left(t = \frac{\pi}{4}\text{seconds}\right) = \sqrt{2}\text{m}$. Please find the value of x at $t = \pi$ seconds. Here, $F_0 = 3\sqrt{2}\text{kg·m/s}^2$, $k = 4\text{kg/s}^2$ and $\Omega = 1\text{s}^{-1}$.

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