

只需填寫答案，不需計算過程，共15題。

- Only answers will be graded. Work need not be shown.
- 6% for each of (1)–(10) and 8% for each of (11)–(15).

(A). Find the interval on which $y = x^3 - 3x^2 - 6x + 1$ is both decreasing and concave upward. Answer. (1).

(B). Find $\frac{dy}{dx} =$ (2) and $\frac{d^2y}{dx^2} =$ (3) at the point $x = 1, y = 1$ of the curve $x^3 + 3x^2y + y^3 = 5$.

(C). Evaluate $\int_4^5 \frac{dx}{x^2 - 3x + 2} =$ (4).

(D). Rotate the region $\{(x, y) : 0 \leq x \leq \pi \text{ and } 0 \leq y \leq \sin x\}$ about the x -axis. Find the volume of the solid obtained. Answer. (5).

(E). Let $g(x)$ be the inverse function of the strictly increasing function $f(x) = x^5 + 3x^3 + 1$. Then $\int_{f(0)}^{f(1)} g(x)dx =$ (6).

(F). The coefficient of x^9 in the Taylor expansion of $\int_0^x e^{-t^2} dt$ is (7).

(G). Find the direction derivative of $f(x, y) = \ln(x^2 + y^2)$ at the point $(1, 2)$ in the direction $\mathbf{u} = (\frac{3}{5}, \frac{4}{5})$. Answer. (8).

(H). Find the plane tangent to the surface $x^3 + y + z^2 - xy^3z^4 = 8$ at the point $(2, 1, 1)$. Answer. (9).

(I). Evaluate $\int_{y=0}^{y=2} \int_{x=y}^{x=2} e^{x^2} dx dy =$ (10).

(J). Let $f(x, y) = \frac{1}{(x^2y^2 - 4x + 3y + 1)^2}$ and $g(u, v) = f(x(u, v), y(u, v))$, where $x(u, v) = u^2 - 3uv + 2v^2$ and $y(u, v) = u^4 + 3uv^3 - 4v^4$. Find $\frac{\partial g}{\partial u}$ at the point $u = 1$ and $v = 1$. Answer. (11).

(K). Find point(s) (12) on the curve $x^3 - y^3 = 1$ farthest from the line $y = x$.

(L). Evaluate $\iint_{\Omega} \left(\frac{y-x}{y+x}\right)^4 dx dy$, where Ω is the region in the first quadrant bounded by the lines $x + y = 1$ and $x + y = 2$. Answer. (13).

(M). Solve the differential equation $\frac{dy}{dt} = 2(1+t)(1+y^2)$ with the initial condition $y(-1) = 0$. Answer. (14).

(N). Solve the differential equation $t \frac{dy}{dt} + 2y = e^t$ with the initial condition $y(1) = 2$. Answer. (15).

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