請依照題號順序作答。所有數字必須化為最簡分數，未依規定者該題不予計分。
除作圖外，答案限用黑色或藍色筆書寫。試題共三大題，第一大題為單選題，每題各佔 5%。 第二大題為多選題，每題各佔 8%。第三大題為計算證明題，該大題佔 18%，共兩題。

一、 單選題（占50分）

（說明: 第 1 題至第 10 題，請於答案卷首頁之選擇題作答區作答，每題 5 分；未作答或答錯者，以零分計算。）

1. 已知一曲線的參數式為 $x = t^2+2t$ 及 $y = t^3+t^2$。下列哪一個選項為前述曲線上 $t = 1$ 的點的切線？
(A) $2x-3y = 0$ (B) $4x-5y = 2$ (C) $4x-5y = 10$ (D) $5x-4y = 7$ (E) $5x-y = 13$。

2. $\int_0^3 (1-x)^{-2}dx$ 為何？
(A) $-3/2$ (B) $-1/2$ (C) $1/2$ (D) $3/2$ (E) 發散。

3. Which of the following integrals gives the length of the graph $y = \sin(\sqrt{x})$ between $x = a$ and $x = b$, where $0 < a < b$?
(A) $\int_a^b \sqrt{x+\cos^2(\sqrt{x})}dx$ (B) $\int_a^b \sqrt{1+\cos^2(\sqrt{x})}dx$ (C) $\int_a^b \sqrt{\sin^2(\sqrt{x})+\frac{1}{4x}\cos^2(\sqrt{x})}dx$
(D) $\int_a^b \sqrt{1+\frac{1}{4x}\cos^2(\sqrt{x})}dx$ (E) $\int_a^b \sqrt{\frac{1+\cos^2(\sqrt{x})}{4x}}dx$。

4. Which of the following integrals represents the area enclosed by the smaller loop of the graph of $r = 1+2\sin\theta$?
(A) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6}(1+2\sin\theta)^2d\theta$ (B) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6}(1+2\sin\theta)d\theta$ (C) $\frac{1}{2}\int_{-\pi/6}^{7\pi/6}(1+2\sin\theta)^2d\theta$
(D) $\int_{-\pi/6}^{7\pi/6}(1+2\sin\theta)^2d\theta$ (E) $\int_{-\pi/6}^{7\pi/6}(1+2\sin\theta)d\theta$。

5. The third-degree Taylor polynomial about $x = 0$ of $\ln(1-x)$ is
(A) $-x-x^2/2-x^3/3$ (B) $1-x+x^2/2$ (C) $x-x^2/2+x^3/3$ (D) $-1+x-x^2/2$
(E) $-x+x^2/2-x^3/3$。

6. Which expression represents the volume of the solid generated when the region between the following two curves

$$y = 6-x^2 \quad \text{and} \quad y = x^2/2$$

over the interval $[0,2]$ is rotated around the x-axis?
(A) $2\pi\int_0^2 y\sqrt{6-y}dy - 2\pi\int_0^2 y^{3/2}\sqrt{2}dy$ (B) $2\pi\int_2^6 y\sqrt{6-y}dy + 2\pi\int_0^2 y^{3/2}\sqrt{2}dy$
(C) $2\pi\int_2^6 y\sqrt{6-y}dy - 2\pi\int_0^2 y^{3/2}\sqrt{2}dy$ (D) $2\pi\int_0^6 y\sqrt{6-y}dy + 2\pi\int_0^2 y^{3/2}\sqrt{2}dy$
(E) $2\pi\int_0^6 y\sqrt{6-y}dy - 2\pi\int_0^2 y^{3/2}\sqrt{2}dy$。

7. The function f satisfies the equation

$$\int_0^{2x} f(t)dt = 6\sin(x)+x.$$

Evaluate $f(\pi/3)$. (A) 6.196 (B) 2 (C) 3.098 (D) 4 (E) 3.055。

見背面

8. The function $g(x)$ is the derivative of $\int_0^x (t^3-5)dt$. What is the derivative of the inverse of $g(x)$ at $x=3$?
(A) 4 (B) 1/4 (C) 12 (D) 1/3 (E) 1/12。

9. 當$\lim_{x\to -a^{1/3}} \sqrt{x^6+2ax^3}-x^3-a$ 存在且其值為 -3。 請問下列哪個選項是正確的?
(A) $a=0$ (B) $a^2=9$ (C) $a=-3$ (D) $a=3$ (E) 以上皆非。

10. 當$\lim_{x\to\infty} \frac{\int_x^\infty t^k \exp(-t^2)dt}{(ax^2+bx+c)\exp(-x^2)} = 3$。 請問下列哪個選項是正確的?
(A) $k=3$ (B) $k=1$ (C) $a=1/6$ (D) $a<b$ (E) $c=3$。

二、多選題 （占 32 分）
（說明: 第 11 題至第 14 題，每題 8 分，請作答於答案卷首頁之選擇題作答區，答錯者倒扣2分，扣至本大題零分為止。）

11. 當 $f(x,y)=x^2+y^2+\exp(x-y)$，且 $F(x,y,z)=f(x,y)-z$。 請問下列哪些選項是正確的?
(A) $\partial f/\partial x+\partial f/\partial y=2x+2y$ (B) $\frac{\partial^2 f}{\partial x^2}-\frac{\partial^2 f}{\partial y^2}=2\exp(x-y)$
(C) ∇F at the point $(1,2,-1)$ is $(1,3,-1)^T$. (D) The tangent plane to the graph of the function $F(x,y)$ at the point $(1,1,3)$ is $3(x-1)+(y-1)-(z-3)=0$.
(E) The minimum value of $f(x,y)$ is 0.

12. 當 $f(x,y)=3x^2y+y^3+6xy$， 請問下列哪些選項是正確的?
(A) $\partial f(x,y)/\partial x=6xy+6y$ (B) $\nabla f(x,y)=(0,0)$ 的解只有四個。
(C) $f(x,y)$的局部極大發生在 $(0,0)$。 (D) $(-2,0)$為$f(x,y)$的鞍點 (saddle point)
(E) $f_{xx}f_{yy}-f_{xy}f_{xy}=6y^2-(x+1)^2$.

13. A region $\mathcal{A}$ is bounded by the hyperbolas (雙曲線) $xy=1$ and $xy=2$, and the curves $xy^2=3$ and $xy^2=4$. The area of $\mathcal{A}$ is denoted by $area_o$. Use the change of variables $u=xy$ and $v=xy^2$, the area of the image of the region $\mathcal{A}$ in the coordinate space determined by (u,v) under the transformation is called $area_t$. 請問下列哪些選項是正確的?
(A) $area_o=\ln 4-\ln 3$ (B) $area_t=\int_1^2\int_3^4 dudv$ (C) $area_o=\int_1^2\int_3^4 1/vdudv$
(D) The Jacobian determinant $\partial(x,y)/\partial(u,v)$ is v. (E) $\partial(u,v)/\partial(x,y)=xy$

14. Let $\mathcal{S}$ denote the solid enclosed by $x^2+y^2+z^2=2z$ and $z^2=x^2+y^2$.
(A) The length of of the curve determined by $\{(x,y,z): x^2+y^2+z^2=2z$ and $z^2=x^2+y^2\}$ is 2π. (B) The volume of $\mathcal{S}$ is less than less than $2\pi/3$.
(C) The volume of $\mathcal{S}$ is equal to π.
(D) 當使用球面坐標系(r,θ,ϕ)，即 $x=r\sin\theta\cos\phi, y=r\sin\theta\sin\phi, z=r\cos\phi$時，$\mathcal{S}$ 的體積等於 $\int_0^{2\pi}\int_0^{\pi/4}\int_0^{2\cos(\phi)}\rho^2\sin(\phi)d\rho d\theta d\phi$。
(E) $\mathcal{S}$ 的表面積大於4π。

接次頁

三、計算題 （佔 18 分）

(說明：本大題共有二題計算證明題，答案務必寫在答案卷上，並於題號欄標明題號 (15, 16) 與子題號((a)、(b))，同時必須寫出演算過程或理由，否則將予扣分。)

15. (8分) Show that the line integral $\int_C 2x\sin(y)dx+(x^2\cos(y)-3y^2)dy$ is independent of path and evaluate the given integral for any path from $(2,\pi/6)$ to $(0,-2)$.

16. (10分) Compute the integral $\oint \frac{-y}{x^2+y^2}dx+\frac{x}{x^2+y^2}dy$.

(a) (5分) Over any closed curve C not enclosing the origin.

(b) (5分) Over the circle of radius a centered at $(0,0)$.

試題隨卷繳回