國立臺灣大學九十五學年度轉學生入學考試試題

題號：19

科目： 微積分(A)　　　　共 1 頁之第 全 頁

請在答案卷上標明題號，按序作答。

一共 5 題，每題 20 分。

1. (i) Find the limit $\lim_{n\to\infty}(\sqrt{n^2+n}-n)$.

   (ii) Determine whether the series converges or diverges： $\sum_{n=1}^{\infty}\frac{(n+1)^n}{n^{n+1}}$.

2. Let $F(x)=\int_0^x \frac{1}{1+t^2}dt+\int_0^{1/x}\frac{1}{1+t^2}dt,\quad x\neq 0$.

   (i) Show that $F(x)$ is constant on the interval $(-\infty,0)$ and constant on $(0,\infty)$.

   (ii) Evaluate the constant value(s) of $F(x)$.

3. (i) Find the integral $\int_0^{\infty} e^{-x^2}dx$.

   (ii) Use (i) to evaluate $\int_{-\infty}^{\infty} e^{-x^2/2}dx$.

4. Let S be the surface $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, and $\overline{N}$ be the outward unit normal vector of S.

   Consider the vector field $\overline{F}=\frac{1}{b^2}xy^2\overline{i}+\frac{1}{a^2}x^2y\overline{j}+\frac{1}{3c^2}z^3\overline{k}$.

   (i) Find the divergence $\nabla\cdot\overline{F}$.

   (ii) Find the surface integral $\iint_S(\overline{F}\cdot\overline{N})dS$.

5. (i) Let $\overline{r}=x\overline{i}+y\overline{j}+z\overline{k}$. Show that $\oint_C \overline{r}\cdot d\overline{r}=0$ for any closed curve C.

   (ii) Let $\overline{F}$ be the vector field $\overline{F}=x\overline{i}+y^2\overline{j}+ze^{xy}\overline{k}$ and S be that part of the surface $z=1-x^2-2y^2$ with $z\geq 0$. Evaluate $\iint_S(\nabla\times\overline{F})\cdot\overline{N}\,dS$, where $\overline{N}$ is the upward unit normal vector of S.

試題必須隨卷繳回