Any device with computer algebra system is prohibited during the exam.

PART 1 : Fill in the blanks.

- Only answers will be graded.
- Each answer must be clearly labeled on the answer sheet.
- 5 points are assigned to each blank.

1. Suppose that $f''(x)$ exists and $\lim_{x\to 1}\dfrac{f(x)-3}{(x-1)^2}=-1$. The equation of the tangent line at $x=1$ is ___(1)___. Let $g(x)=2^{-f(x)}$. Then $g''(1)=$ ___(2)___.

2. Suppose that $f(u)$ is continuous and $f(u)>0$ for all u. Let $g(x)=\int_0^x \left(t\int_{t^2}^1 f(u)\,du\right)dt$. Then $g(x)$ obtains local maximum at $x=$ ___(3)___. $g(1)=\int_0^1 h(u)f(u)\,du$, where $h(u)=$ ___(4)___.

3. (a) Evaluate $\int_0^{\frac{1}{2}}\dfrac{1+x}{1+x^3}dx=$ ___(5)___.

   (b) Write $\int_0^{\frac{1}{2}}\dfrac{1+x}{1+x^3}$ as the sum of the series $\sum_{n=0}^{\infty}\dfrac{a_n}{8^n}$, where $a_n=$ ___(6)___.

   (c) Use a partial sum of the series from (b) with least terms to estimate the number π within error 10^{-3}. Answer = ___(7)___.

4. The function $f(x,y,z)$ has continuous partial derivatives. Assume that point $(1,2,-1)$ lies on the level surface $S: f(x,y,z)=5$ and the tangent plane of S at $(1,2,-1)$ is $-x+y+3z=-2$. Assume that at $(1,2,-1)$ along the vector $\mathbf{u}=(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{-1}{\sqrt{3}})$ the directional derivative $D_{\mathbf{u}}f$ is $2\sqrt{3}$.

   (a) At $(1,2,-1)$, $\nabla f=$ ___(8)___.

   (b) Use linear approximation to estimate $f(1.01,1.9,-0.97)$. Answer = ___(9)___.

5. Evaluate the double integral

$$\int_0^{\frac{1}{\sqrt{2}}}\int_{\sqrt{1-x^2}}^{\sqrt{3-x^2}}\frac{x}{1+x^2+y^2}dydx+\int_{\frac{1}{\sqrt{2}}}^{\sqrt{\frac{3}{2}}}\int_x^{\sqrt{3-x^2}}\frac{x}{1+x^2+y^2}dydx=\underline{\quad(10)\quad}.$$

PART 2 :

- Solve the following problems. You need to write down your reasoning.
- 10 points are assigned to each problem.

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1. Solve the differential equation $y'(t) = y(t)(1 - (\frac{y(t)}{M})^2)$, $y(0) = y_0$, where $M > 0$ is a constant and $0 < y_0 < M$.

   (a) First compute $\int \frac{1}{x(1-(\frac{x}{M})^2)}dx$.

   (b) Solve the differential equation.

   (c) Find $\lim_{t\to\infty} y(t)$ and $\lim_{M\to\infty} y(t)$.

2. Suppose that the production P depends on the amount L of labor used and the amount K of capital invested, $P = L^a K^{1-a}$ for some constant a, $0 < a < 1$. Assume that the cost of a unit of labor is 20 thousand dollars and the cost of a unit capital is 50 thousand dollars. If a company can spend C thousand dollars as its budget, then the company seeks the maximum production subject to the constraint $20L + 50K = C$.

   (a) Solve for the maximum production, P_{max}, in terms of C and a.

   (b) Compute $\frac{\partial P_{max}}{\partial a}$ when $a = \frac{1}{2}$, and $C = 1000$.

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