※ Please show the detailed calculation process for the questions whenever appropriate.

1. (13%) State the definition of σ-algebra. Explain the concept of σ-algebra.

2. (13%) Explain the concepts of probability measure and probability space.

3. (10%) Suppose we are playing draw poker. We are dealt (from a well shuffled deck) 5 cards which contain 4 spades and another card of a different suite. We decide to discard the card of a different suite and draw one card from the remaining cards to complete a flush in spades (all 5 cards spades). Determine the probability of completing the flush.

4. (10%) Let *X* have the pdf $f(x) = 3x^2,\ 0 < x < 1$, zero elsewhere. Consider a random rectangle whose sides are *X* and (*1-X*). Determine the expected value of the area of the rectangle.

5. (10%) If $f(x) = \frac{1}{2}, -1 < x < 1$, zero elsewhere, is the pdf of the random variable *X*, find the pdf of $Y = X^2$.

6. (10%) If the random variable *X* has a Poisson distribution such that $P(X = 1) = P(X = 2)$, find $P(X = 4)$.

7. (10%) Let *X* and *Y* have a bivariate normal distribution with parameters $\mu_1 = 5,\ \mu_2 = 10,\ \sigma_1^2 = 1,\ \sigma_2^2 = 25$, and $\rho > 0$. If $P(4 < Y < 16|X = 5) = 0.954$, determine ρ.

8. (10%) Let $\overline{X}$ denote the mean of a random sample of size 128 from a gamma distribution with $\alpha = 2$ and $\beta = 4$. Approximate $P(7 < \overline{X} < 9)$.

9. (14%) Let $X_1, \ldots, X_n$ be iid $n(\mu, \sigma^2)$, where σ^2 is known. Show that the sample mean, $T(X) = \overline{X} = (X_1 + \cdots + X_n)/n$, is a sufficient statistic for μ.

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