A. 腳踏車動力學: 腳踏車通常由兩個機械系統組成，包含車輪機械系統以及踏板機械系統，你將被要求研究這兩個機械系統:

1. **車輪機械系統 (25%):** 請分析圖 1 的腳踏車傳動系統，其機械自由體圖如圖 2 表示，其中 T、J、B、θ、ω 分別表示扭矩、轉動慣量、阻尼係數、旋轉角度，速度，下標 pedal 與 load 分別代表踩踏輸入與後輪負載，Kc 為鍊條剛性係數，Ri 及 Ro 分別代表輸入端齒輪數及負載端齒輪數。
   1.1 (10%): 請推導腳踏車車輪傳動系統動態方程式 dynamic equations (共兩條方程式):
   1.2 (15%): 假設是在逆風模式，T_{pedal} 是一個常數，但是 $T_{load}=\rho V_{wind}^2 A$ (風阻)是一個隨速度上升的負載，其數值如下
   T_{pedal} 為 31.58Nm、J_{pedal} 為 6.32×10^{-1}、J_{wheel} 為 6.34×10^{-2}、B_{pedal} 為 8.3×10^{-3}、B_{wheel} 為 9×10^{-2}、θ_{pedal} 為 0°、θ_{wheel} 為 0°、Ri 為 34 齒、Ro 為 14 齒、Kc 為∞(剛性鍊條)、ρ 為 1.2258、A 為 $0.645m^2$、V_{wind} 為 10km/h
   請推導 ω_{wheel} 解析解(包含變數與數值)及劃出縱軸 ω_{wheel} 與時間的示意圖

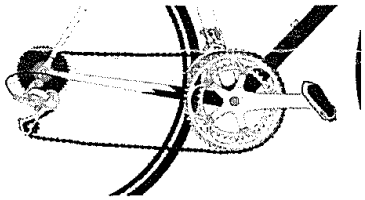

圖 1 腳踏車車輪傳動系統

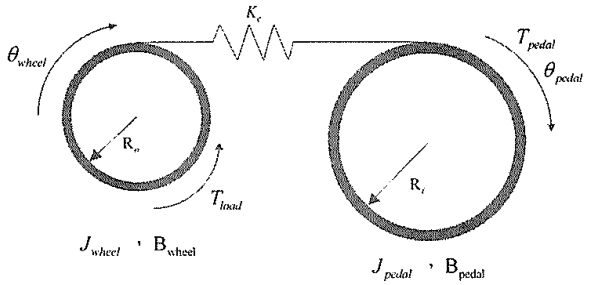

圖 2 車輪傳動系統自由體圖

見背面

2. **踏板機械系統(25%):** 圖 3 說明腳踏車的踏板詳細機械系統，騎乘者的踩踏力分為左右腳 F_{right} 和 F_{left}，踏板永遠平行於地面，踩踏力會與踏板軸力矩 T_{pedal} 有直接對應關係
   2.1 (10%)假設左腳踩踏力 F_{left} 與右腳踩踏力 F_{right} 如圖 4 表示，不考慮上題的車輪負載 T_{load}，請用數學式說明踏板軸力矩 T_{pedal} 與踩踏力關係，並且畫出踏板軸力矩 T_{pedal} 與踏板角度 θ_{pedal} 大概關係圖(至少劃出一個週期 0~360 度)
   2.2 (15%)假設踏板軸力矩 T_{pedal} 與踏板角度 θ_{pedal} 的關係為一常數如圖 5，請用數學式回推左右腳踩踏力 F_{right} 和 F_{left} 與踏板角度 θ_{pedal} 關係式，並且畫出 F_{right} 和 F_{left} 與踏板角度 θ_{pedal} 大概關係圖(至少劃出一個週期 0~360 度)

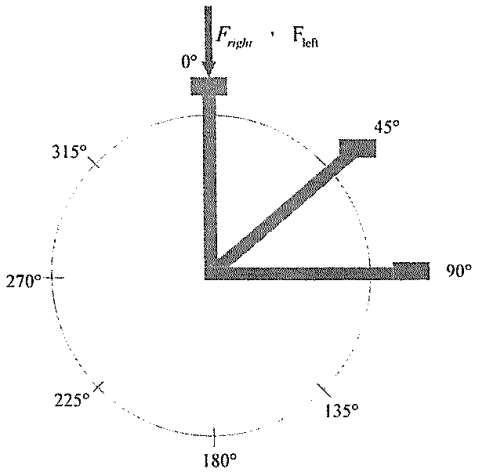

圖 3 腳踏車的板機械系統

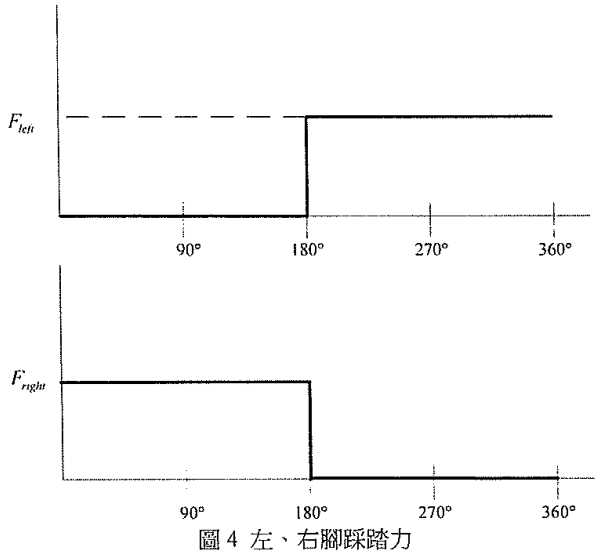

圖 4 左、右腳踩踏力

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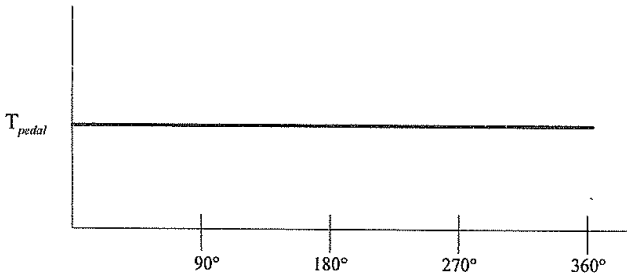

圖 5 踏板軸力矩 T_{pedal} 與踏板角度 θ_{pedal} 之關係

B. (25%) You are requested to participate in a court case as an expert witness. As illustrated in Fig. 6, a spacecraft is known to have unfortunately exploded into three pieces, A, B, and C. m_A = 1000 kg, m_B = 500 kg, and m_C = 500 kg. Telemetry data showed that the positions of the satellite with respect to a known reference position O at a particular time were $R_{A/O} = (400\,\boldsymbol{i} - 1000\,\boldsymbol{j})$ m, $R_{B/O} = 1000\,\boldsymbol{j}$ m, and $R_{C/O} = (800\,\boldsymbol{i} + 1000\,\boldsymbol{j})$ m. The respective velocities were $V_A = -500\boldsymbol{j}$ m/s, $V_B = (-400\boldsymbol{i} + 500\boldsymbol{j})$ m/s, and $V_C = (400\boldsymbol{i} + 500\boldsymbol{j})$ m/s. The engineer of the spacecraft company claims that a single explosion caused the described configuration, whereas the lawyer representing the insurance company contends that at least two separate explosions were responsible. Who is correct? (You have to describe detailed reasons and calculation results to support your answer. 0 point will be given if no explanation is provided.)

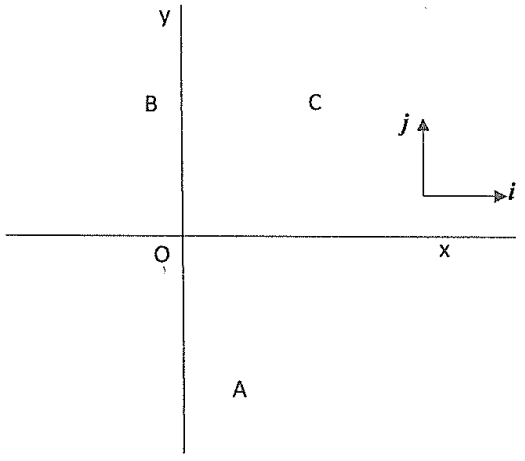

Fig. 6

見背面

C. (25%) As shown in Fig. 7, a 2 m long, 30 kg slender bar is constrained to move in the illustrated guide. Point B is able to move in the vertical direction, and A moves in the horizontal direction. At the instant shown, the spring is uncompressed, and the speed of B is 2 m/s downward. The vertical guide provides a constraint force of 100N for B, and the spring constant is k = 100 N/m.

1. (3%) What assumption is needed so that the system depicted in Fig. 7 can be treated as a conservative system?
2. (8%) Draw the free-body diagram for the system with the assumption stated above.
3. (14%) Determine the velocity of A and the total kinetic energy of the system when B contacts the horizontal guide.

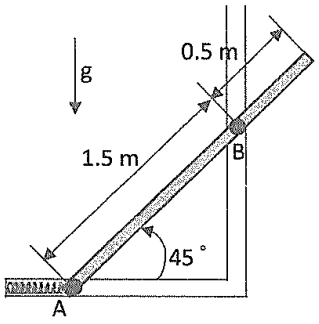

Fig. 7

試題隨卷繳回