財金數學:請詳細列出計算過程

1. 設f(x,y) $= x^3 + 3xy^2 + 6xy - 9x$，求 f 的極值與鞍點。(10 分)
2. 求算$\int_{-\infty}^{0} e^{-x^2} dx$ =? (10 分)
3. 試利用全微分估計$\sqrt{(5.013)^2 + (11.974)^2}$ (8 分)
4. 假設 3 個月後的景氣狀態有好、普通、壞三種狀態，有一市值 100 元的股票，在上述三種狀態下，3 個月後的股價分別為 125, 100, 80 元。另外，以該股票為標的資產，3 個月後到期執行價 110 元的歐式買權(European call option)市價為 5 元。3 個月後到期面額 100 元的無風險零息債券市價 99 元。 (12 分)

   (a) 請以上述 3 種資產複製 3 個月後到期執行價 95 元的歐式買權(European call option)。在無套利的情況下，此買權的價值為何?

   (b) 請以上述 3 種資產複製 3 個月後到期執行價 110 元的歐式賣權(European put option)。在無套利的情況下，此賣權的價值為何?

5. 假設 3 個標準常態分配變數x_1, x_2, x_3的變異數-共變數矩陣(variance-covariance matrix)為

   $\begin{bmatrix} 1 & 0.8 & 0.6 \\ 0.8 & 1 & 0.72 \\ 0.6 & 0.72 & 1 \end{bmatrix}$，請利用 Cholesky decomposition 將上述 3 個標準常態分配變數分解為 3 個獨立標準常態分配變數$\varepsilon_1, \varepsilon_2, \varepsilon_3$的線性組合。(10 分)

財務管理：填充題及是非題。請勿直接將答案填寫於本試題紙上，答案應填寫於答案紙上。如有需要，請將答案四捨五入至小數點第四位。

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6. A foundation announces that it will be offering one NTU scholarship every year for an indefinite number of years. The first scholarship is to be offered exactly one year from now. When the scholarship is offered, the student will receive $20,000 annually for a period of four years, beginning from the date the scholarship is offered. This student is then expected to repay the principal amount received ($80,000) in 10 equal annual installments, interest-free, starting one year after the expiration of her scholarship. This implies that the foundation is really giving an interest-free loan. The current interest is 6% for all maturities and is expected to remain unchanged.
   (a) What is the PV of the first scholarship? _____ (4 points)

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(b) The foundation invests a lump sum to fund all future scholarships. Determine the size of the investment today._____ (4 points)

7. You manage a pension fund that will provide retired workers with lifetime annuities. You determine that the payouts of the fund are level perpetuities of $1 million per year. The interest rate is 10%. You plan to fully fund the obligation using 5-year maturity and 20-year maturity zero-coupon bonds.
   (a) How much *market value* of each of the zeros will be necessary to fund the plan if you desire an immunized position?_____ (4 points)
   (b) What must be the *face value* of the two zeros to fund the plan?_____ (4 points)

8. Assume perfect markets: no transaction costs and no constraints. The one-month risk-free interest rate will remain constant over a six-month period. Two futures contracts are traded on a financial asset without payouts: a three-month (futures price $F(t, t + 3)$) and a six-month (futures price $F(t, t + 6)$) contract. You can observe that $F(t, t + 3) = \$120$ and $F(t, t + 6) = \$122$. What is the spot price of the underlying asset at time t?_____ (4 points)

9. Stock XYZ is worth S = $80 today. Every 6 months the stock price goes either up by $u = 1.3$ or down by $d = 0.8$. The riskless rate is 6% APR with semiannual compounding. The stock pays no dividends.
   (a) Compute the price of a European call with a maturity of 1 year and a strike price of $X = \$95$. _____ (3 points)
   (b) Compute the price of an American call with a maturity of 1 year and a strike price of $X = \$95$. _____ (3 points)
   (c) Compute the price of a European put with a maturity of 1 year and a strike price of $X = \$95$. _____ (3 points)

10. The price of the stock of NewWorld Chemicals Company is $80. The standard deviation of NewWorld's stock returns is 50%. The 1-year interest rate is 6%.
   (a) What should be the price of a call on one share of NewWorld with a maturity of 1 year and strike price of $85? Use the Black-Scholes formula. _____ (3 points)
   (b) What should be the price of a put on one share of NewWorld with the same maturity and strike price? _____ (3 points)

11. Your current portfolio consists of three assets, the common stock of IBM and GM combined with an investment in the riskless asset. You know the following about the stocks ($\rho_{i,j}$ denotes the correlation between asset i and asset j, and M denotes the market portfolio):
   $\rho_{IBM,M} = 0.60$ $\rho_{GM,M} = 0.80$
   $\sigma^2_{IBM} = 0.0900$ $\sigma^2_{GM} = 0.0625$
   You also have the following data about the market portfolio M and the riskless asset F:
   $\bar{r}_M = 0.13$
   $r_F = 0.04$
   $\sigma^2_M = 0.04$
   Suppose that individuals can borrow and lend at r_F and that the Capital Asset Pricing Model (CAPM) describes expected returns on assets. You have $200, 000 invested in IBM, $200,000 invested in GM, and $100,000 invested in the riskless asset.
   (a) What is the expected rate of return on IBM stock? _____ (2 points)
   What is the expected rate of return on GM stock? _____ (2 points)

接次頁

(b) Assume that the correlation between IBM and GM, $\rho_{IBM,GM}$, is 0.40. What is the variance of your portfolio?_____ (2 points) What is its beta, $\beta_{p,M}$? _____ (2 points)

(c) Suppose that you can also invest in the market portfolio. Find an efficient portfolio that has *the same standard deviation* as your portfolio, but has the highest expected rate of return possible. What is the expected rate of return on this portfolio?_____ (4 points)

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12. True or false: You can construct a portfolio with a beta of 0.75 by investing 0.75 of the investment budget in bills and the remainder in the market portfolio. _____ (1 point)
13. True or false: Within an industry, holding business risk and financial leverage constant, differences in firms' P/Es depend only on differences in rates of asset growth. _____ (1 point)
14. True, false: The term structure of interest rates is always upward sloping because bonds with longer maturities are riskier and earn higher returns. _____ (1 point)

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