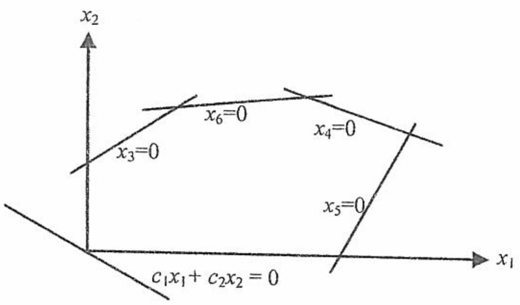
國立臺灣大學99學年度碩士班招生考試試題

Problem 1, 30%: Consider the graphical representation of the following Linear Programming. Answer the following questions.

$$\max\ c_1x_1 + c_2x_2$$

s.t.

$$a_{11}x_1 + a_{12}x_2 + x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + x_4 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + x_5 = b_3$$
$$a_{41}x_1 + a_{42}x_2 + x_6 = b_4$$
$$x_i \geq 0,\ i = 1, \ldots, 6$$



(a)(6%) The initial basic feasible solution (BFS) is at origin. If we always choose the variable with the largest rate of improvement (the steepest ascent method) as an entering variable, then clearly specify the basic variables of the initial BFS and each succeeding basic variables until the optimal point is reached.

(b)(7%) What is the minimum decrement of c_1 so that alternative optimal solutions including the optimal point obtained in (a) exist?

(c)(7%) Find the maximum value for b_2 for which the basis found in (a) remains optimal.

(d)(5%) Assume that **only** one of constraints 2 and 4 ($a_{21}x_1 + a_{22}x_2 + x_4 = b_2$, $a_{41}x_1 + a_{42}x_2 + x_6 = b_4$) has to be satisfied, please re-formulate the problem.

(e)(5%) Continued with (d), what is the optimal objective value for the new problem?

Problem 2, 20%: Consider the following transportation problem in which the objective is to minimize the total cost. Answer the following questions.

		Destination				
		1	2	3	4	Supply
Source	1	3	4	9	8	10
	2	4	6	5	9	35
	3	8	5	7	6	20
Demand		20	20	10	15	

(a)(3%) What is the number of basic variables?

(b)(5%) Let x_{ij} be the number of goods shipped from source i to destination j. Let x_{11}=10, x_{21}=10, x_{22}=15, x_{23}=10, x_{32}=5, and x_{34}=15 be the current solution. Find the variable (or cell) with the most negative value of $c_{ij} - u_i - v_j$ (The c_{ij} represents the cost per unit shipped from source i to destination j. The u_i and v_j are dual variables associated with supply and demand constraints.)

(c)(4%) The shipments (or the allocation of goods) of which variables should be decreased and which one will become a nonbasic variable?

(d)(3%) Continued with (c), obtain the new solution.

(e)(5%) Continued with (d), does this new solution satisfy the primal and dual feasibility? Justify your answer.

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Problem 3, 16%: A monopolist sells a single product on the market. If the price of this product is p, then the demand for this product is q units and $q=(30-p)$. For the monopolist, the cost of manufacturing q units is $(5+2q)$. (p and q may not be integers in this problem.) To maximize profit,

(a)(4%) Formulate a non-linear profit maximization problem for the monopolist?

(b)(5%) Does this non-linear optimization problem has a unique solution? Explain your answer.

(c)(7%) How many units should the monopolist sell on the market to maximize profit?

Problem 4, 17%: Suppose the entire cola industry produces only three types of colas.

✧ Given that a person last purchased *Cola 1*, there are a 90% chance that her next purchase will be *Cola 1* and a 10% chance that her next purchase will be *Cola 2*.

✧ Given that a person last purchased *Cola 2*, there are an 80% chance that her next purchase will be *Cola 2* and a 20% chance that her next purchase will be *Cola 1*.

✧ Given that a person last purchased *Cola 3*, there are a 70% chance that her next purchase will be *Cola 2* and a 30% chance that her next purchase will be *Cola 3*.

Let X_n denotes the type of cola purchased by a person in her n^{th} purchase. Then, the purchase of this person can be modeled as a Markov chain.

(a)(4%) Find the one-step transition probability matrix for this Markov chain?

(b)(4%) If a person is currently a *Cola 3* purchaser, what is the probability that she will purchase *Cola 1* two purchases from now?

(c)(4%) Determine which states are transient and which are recurrent. Explain your answer.

(d)(5%) Let X_0 = *Cola 3*. Find the limiting probability $\lim_{n\to\infty} P\left(X_n = Cola\ 1 \middle| X_0 = Cola\ 3\right)$. Explain your answer.

Problem 5, 17%: A company produces and sells a single type of airplane engine. The demands for the current and the following three months are 2, 1, 1, and 1 engines, respectively. Demand for a specific month can be fulfilled by either inventory on hand or production of the month. All demands must be fulfilled before the end of each month, and no backorders are allowed.

✧ Due to warehouse capacity limit, the company cannot store more than 2 engines at the end of each month. For unsold engines at the end of a month, inventory holding costs are $1 per engine.

✧ The company cannot produce more than two engines within each month. The costs for producing 0, 1, and 2 engines are $10, $17, and $20, respectively.

✧ This company has 2 engines on hand at the beginning of the current month. The manager wants to have 1 engine on hand by the end of the last month.

The company wants find a production plan that minimizes total costs for the current and the following three months. This production plan must satisfy all constraints.

(a)(8%) Model this finite horizon production planning problem using dynamic programming. Clearly explain your answer.

(b)(9%) Find an optimal production plan and optimal total costs by solving the dynamic programming problem.

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