1. (20 %) Consider three same circular small cylinders confined by a big semi-circular cylinder as shown in Fig. 1. The big cylinder is fixed on the ground. Denote the radii of the small and big cylinders by r and R, respectively. Neglect all the friction effects at the interfaces. Determine the relations between r and R such that the whole system is kept in static equilibrium as shown.

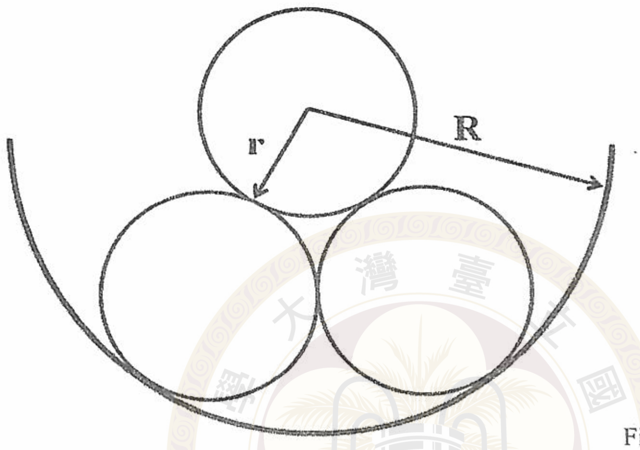

Fig. 1

2. (25 %) Consider a projectile motion of mass m near the ground surface in two dimensions. In addition to a gravitational force due to the constant field g, assume that the force caused by the air resistance is directly proportional to the projectile's velocity with the proportional coefficient k. Let the muzzle velocity of the projectile be V_0 and the angle of elevation be θ, as shown in Fig. 2. Calculate the displacement and velocity of the projectile as functions of time. Compare your results with those without the air resistance.

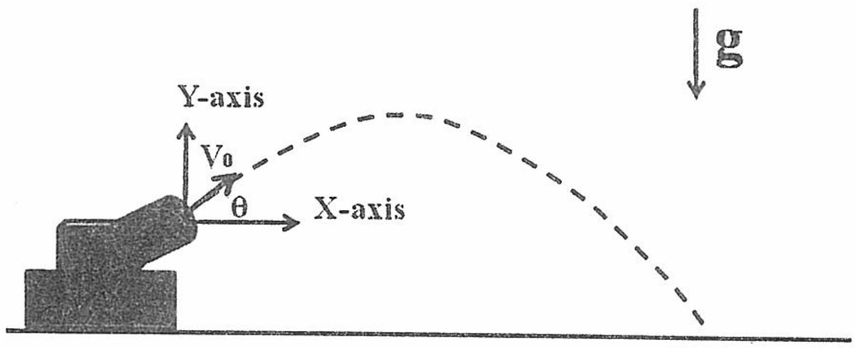

Fig. 2

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3. (25 %) A massless, inextensible string passes over a pulley which is at fixed distance above the floor, cf. the attached figure (Fig. 3). A bunch of bananas of mass m is attached to one end A of the string. A monkey of mass M is initially at the other end B. The monkey climbs the string, and his displacement with respect to the end B is a given function of time $d(t)$. The system is initially at rest, so that the initial conditions are $d(0)=0,\ \dot{d}(0)=0$. Derive the equation of motion governing the height of the monkey above the floor. In the special case that $m=M$, show that the bananas and the monkey rise through equal distances so that the vertical separation between them is constant.

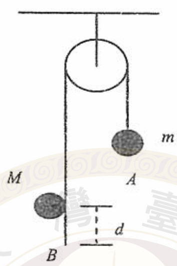

Fig. 3

4. (30 %) Consider the rotational motion of a torque-free rigid body with its center of mass being fixed.

(1) (10 %) Let I_1, I_2, I_3 denote the three principal axis of the rigid body, and $\omega_1, \omega_2, \omega_3$ be the components of the angular velocity of the body with respect to the frame formed by the principal axis. Show that the motion of the body is governed by the following Euler equation of motion:

$$I_1\dot{\omega}_1=(I_2-I_3)\omega_2\omega_3,$$
$$I_2\dot{\omega}_2=(I_3-I_1)\omega_3\omega_1,$$
$$I_3\dot{\omega}_3=(I_1-I_2)\omega_1\omega_2,$$

(2) (8 %) A steady motion of the body refers to an equilibrium state of the Euler's equation. Show that the motion of $\omega_1=c,\ \omega_2=0,\ \omega_3=0$, where c is a constant, is a steady motion.

(3) (6 %) Consider the perturbed motion of the steady motion $\omega_1=c,\ \omega_2=0,\ \omega_3=0$, expressed as $\omega_1=c+\xi_1,\ \omega_2=\xi_2,\ \omega_3=\xi_3$. Ignoring the product terms, derive the linear equation for the perturbations ξ_1, ξ_2, ξ_3.

(4) (6 %) The steady motion is unstable if the set of linear equations of perturbations has eigenvalues in the right-half complex plane. Show that if $I_2<I_1<I_3$, the corresponding steady motion $\omega_1=c,\ \omega_2=0,\ \omega_3=0$, i.e. rotating about the intermediate principal axis, is unstable.

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