1. **(10%)** Find the general solution of the differential equation:

$$\frac{dy}{dx} = \frac{-y}{1+ye^x}$$

2. **(15%)** Tank 1 initially contains 10 m^3 of aqueous sugar solution in which 15 kg of sugar is dissolved. Tank 2 initially contains 10 m^3 of pure water. Pure water flows into tank 1 at the rate of 2 m^3/hr. The solution in tank 1 flows into tank 2 at the rate of 2 m^3/hr, and meantime the solution in tank 1 is also flushed away at the rate of 1 m^3/hr. The solution in tank 2 is pumped back into tank 1 at the rate of 1 m^3/hr, and the solution in tank 2 is also pumped into a third tank at the rate of 1 m^3/hr. The solution in each tank is kept uniform by stirring. Determine the amount of sugar in tank 1, $x_1(t)$, and also that in tank 2, $x_2(t)$, at any time $t \geqq 0$.

3. **(15%)** Use the Laplace transform method to solve the problem:

$$\frac{d^2y}{dt^2} - 2t\frac{dy}{dt} + 4y = 2, \quad t \geq 0$$

$$y(0) = \frac{dy(0)}{dt} = 0$$

Hint: If the Laplace transform of $f(t)$ is $L\{f(t)\} = F(s)$, then $L\{t\,f(t)\} = -\frac{dF(s)}{ds}$

4. **(10%)** Find the eigenvalues and the corresponding eigenfunctions of the Sturm-Liouville problem:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + (1+\lambda)y = 0$$

$$y(0) = y(1) = 0$$

5. **(15%)** A certain three-dimensional curve is described by $[\cos(t), \sin(t), t]$ with t being the parameter. Determine the following quantities of the curve: (a) the unit tangent vector, (b) the curvature, (c) the normal vector.

6. **(15%)** State the following theorems in vector analysis: (a) Green's theorem, (b) the divergence theorem of Gauss, (c) the integral theorem of Stokes. Define explicitly all the symbols used.

7. **(20%)** Solve the partial differential equation and the associated boundary conditions below for $u=u(x,y)$:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < a, \quad 0 < y < b$$

$$u(0,y) = 0, \quad 0 < y < b$$

$$u(a,y) = f(y), \quad 0 < y < b$$

$$u(x,0) = 0, \quad 0 < x < a$$

$$u(x,b) = g(y), \quad 0 < x < a$$

試題隨卷繳回