- 本試題共 8 大題, 合計 100 分。
- 請依題號依序作答。
- 請詳述理由或計算推導過程, 否則不予計分。

1. (20 分) Let $\{X_i\}_{i=1}^n$ be a random sample. Assume $E(X_1) = \mu$ and $Var(X_1) = \sigma^2$ are unknown. Suppose we decide to test $\mu = 0$ against $\mu \neq 0$ by rolling a fair ten-sided die and rejecting the null if the number 1 turns up.
   (a) What is the size of this test?
   (b) What is the power of this test?
   (c) In practice, would you use the test proposed above?
   (d) Let $D \sim \text{Bernoulli}(p)$. Find out $E(X_1 D) =$?

2. (10 分) Let $\{X_i\}_{i=1}^n \sim^{i.i.d.} f(x;\theta)$, where
$$f(x;\theta) = \theta(1-\theta)^x, \quad x = 0, 1, 2, \cdots$$
   (a) Using the method of moments to find the estimator for θ (denoted by $\hat{\theta}_n$).
   (b) Find the asymptotic distribution of $\hat{\theta}_n$.

3. (10 分) Let X, Y and Z be continuously distributed random variables. The probability density function is denoted by $f(\cdot)$.
   (a) Show that $f(y, z|x) = f(y|z, x)f(z|x)$.
   (b) Y and Z are said to be conditionally independent given X if and only if $f(y, z|x) = f(y|x)f(z|x)$. Show that if Y and Z are independent given X, then $E(Y|Z, X) = E(Y|X)$.

4. (10 分) Let $X \sim N(\mu_1, \sigma_1^2)$, $Y \sim N(\mu_2, \sigma_2^2)$, and $U \sim \text{Uniform}(0, 1)$ such that U and (X, Y) are independent. For a given constant $\alpha \in (0, 1)$, define the new random variable
$$Z = \begin{cases} X & \text{if } U \leq \alpha, \\ Y & \text{otherwise.} \end{cases}$$
   Let $\Phi(\cdot)$ denote the distribution function of a standard normal random variable. Find the distribution function of Z.

5. (15 分) **True, false, or uncertain?** Evaluate the following statements with explanations.
   (a) If the error terms are heteroscedastic, then the ordinary least squares (OLS) estimator is not consistent.

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(b) There are four binary variables for educational attainment and each observation falls into one and only one category. If all four binary variables are included as regressors, the regression will fail because of perfect multicollinearity even if the intercept is omitted.

(c) The average scores for Taiwanese and Korean college students who took TOEIC (Test of English for International Communication) examination are 577 and 621, respectively. This suggests that the mean English ability of Taiwanese college students are inferior to that of Korean college students.

6. (10 分) Suppose you have estimated $\hat{Y} = 4 + 2X + 2D$ where Y is earnings, X is years of working experience, and D is 0 for females and 1 for males.

   (a) If we were to rerun this regression with the dummy variable D defined as 1 for females and 2 for males, what results would we get?

   (b) If the dummy variable D were defined as -1 for males and +1 for females, what results would we get?

7. (10 分) If an error were generated in the regression model

$$Y_i = \beta_0 + \beta_1 X_i + u_i, \quad \mathrm{E}(u_i|X_i) = 0$$

by entering data that always contained the same typographical error,

   (a) $\tilde{X}_i = X_i + a$, or

   (b) $\tilde{X}_i = bX_i$,

where a and b are contants. What effects would each of these have on the probability limit of the lease squares estimator $\hat{\beta}_1$? (Note: If $\hat{\beta}_1$ converges in probability to c as $n \to \infty$, then the probability limit of $\hat{\beta}_1$ is c.)

8. (15 分) The model is

$$Y_i = \beta X_i + u_i, \ \mathrm{E}(u_i|X_i) = 0, \ \ \mathrm{E}(u_i^2|X_i) = \sigma^2$$

where X_i is a scalar. Consider the estimator

$$\tilde{\beta} = \frac{\bar{Y}}{\bar{X}} = \frac{\frac{1}{n}\sum_{i=1}^{n} Y_i}{\frac{1}{n}\sum_{i=1}^{n} X_i}.$$

Assume that X_i and u_i have finite fourth moments and that $\{Y_i, X_i\}$ is a random sample.

   (a) Find $\mathrm{E}(\tilde{\beta}|X)$ and $\mathrm{Var}(\tilde{\beta}|X)$.

   (b) Is $\tilde{\beta}$ consistent? Does your answer need any additional assumptions?

   (c) Without imposing any additional assumptions, is $\tilde{\beta}$ necessarily less efficient than $\hat{\beta}_{OLS}$, the OLS estimator of β?

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