國立臺灣大學98學年度碩士班招生考試試題

※ 注意：「第一大題選擇題考生應作答於『答案卡』，並請用 2B 鉛筆作答。」

第一大題－多重選擇題：

＊每題有一個以上正確選項。每一選項答錯(應選而未選或不應選而選)扣2.5分。每題未作答或答錯兩個以上選項者，該題以零分計。

1. (5%) Consider the following wireless communication technologies, which support(s) mobile data services over wide range area?
(A) Bluetooth (B) WiMAX (C) UWB (D) LTE (E) WiFi.

2. (5%) Orthogonal Frequency Division Multiplexing (OFDM) is widely used in wireless communication systems nowadays. Compared to single carrier systems, OFDM has the advantage(s) of:
(A) Bandwidth efficient (B) Intersymbol interference eliminated (C) Less sensitive to carrier frequency offset (D) Less sensitive to symbol timing error (E) Low peak-to-average power ratio.

3. (5%) Consider the following statements:

   i. The low SNR performance of a modulation scheme on an AWGN channel is characterized by the minimum distance of the signal constellation.

   ii. For an M-ary modulation scheme, the minimum bit error probability decision rule is the same as the minimum symbol error probability decision rule as long as the noise is AWGN and all symbols are equally likely to be transmitted.

   iii. As M increases, accurate phase synchronization becomes more important for M-PSK.

   iv. AWGN noise model is often used to model thermal noise.

   Which of the statements above is(are) false:
   (A) i (B) ii (C) iii (D) iv (E) None of the above.

4. (5%) Consider two different bit mapping schemes for the same 16-QAM signal constellation. One is Gray-mapped, and the other is not. Compare the performance of the two schemes under the same **high** SNR. Select the **correct** statement(s) from the following:
(A) The Gray-mapped 16-QAM has lower bit error probability than that of the non-Gray-mapped 16-QAM (B) The Gray-mapped 16-QAM has lower symbol error probability than that of the non-Gray-mapped 16-QAM (C) The two schemes have the same bit error probability (D) The two schemes have the same symbol error probability (E) None of the above.

5. (5%) Consider a simple channel code which consists of 4 possible codewords: $\mathbb{C} = \{00000, 11000, 00111, 11111\}$. A codeword is randomly chosen from $\mathbb{C}$ and transmitted through a binary symmetric channel (BSC) with bit error probability $p = 0.55$. From the BSC output, the receiver receives 00001. If the receiver performs hard decision decoding following MAP rule, what will the decoded result be?
(A) 00000 (B) 00001 (C) 11000 (D) 00111 (E) 11111

6. (5%) Consider the performance of two systems, U and R. System U does not apply any channel coding and the data bit is directly transmitted using BPSK modulation; while system R encodes the data bit using (5,1) repetition code ($\mathbb{C} = \{00000, 11111\}$) and then transmits the 5 coded bits using BPSK modulation. *Assume that both systems operate under the same* $\frac{E_b}{N_0}$, *where* E_b *denotes the energy per* data bit. Consider the following statements:

   i. The decoding error probability of system R using the optimal **hard-decision** decoding is lower than the bit error probability of the **uncoded** system U.

   ii. The decoding error probability of system R using the optimal **hard-decision** decoding is equal to the bit error probability of the **uncoded** system U.

   iii. The decoding error probability of system R using the optimal **soft-decision** decoding is lower than the bit error probability of the **uncoded** system U.

   iv. The decoding error probability of system R using the optimal **soft-decision** decoding is equal to the bit error probability of the **uncoded** system U.

   Which of them is(are) **correct**?
   (A) i (B) ii (C) iii (D) iv (E) None of the above.

7. (5%) *Consider a sequence* $x[n]$ *whose Fourier transform* $X(e^{jw})$ *has the property that*

$$X(e^{jw}) = 0 \quad \text{for} \quad \frac{2\pi}{9} \leq |w| \leq \pi.$$

   Then $x[n]$ may be sampled without the possibility of aliasing for which of the following sampling periods? (A) 2 (B) 4 (C) 6 (D) 8 (E) None of the above.

見背面

8. (5%) Choose the correct statement(s) from the following:

   (A) The continuous-time system with output $y(t)$ and input $x(t)$ as described by $y(t) = x(\sin(t))$ is causal.

   (B) The continuous-time system with output $y(t)$ and input $x(t)$ as described by $y(t) = \cos(t)x(t)$ is time varying.

   (C) Let $x[n]$ and $y[n]$ be two discrete-time signals, where $y[n] = x[2n]$. If $y[n]$ is periodic, then $x[n]$ is periodic.

   (D) Let $x[n]$ and $y[n]$ be two discrete-time signals, where $x[n]$ is an odd signal and $y[n]$ is an even signal. Then $z[n] = x[n]y[n]$ is an odd signal.

   (E) None of the above.

9. (5%) *Consider two sequences*

$$x[n] = \begin{cases} \alpha^n, & 0 \le n \le 6 \\ 0, & \text{otherwise} \end{cases}, \quad \text{and} \quad y[n] = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{otherwise} \end{cases}.$$

   Let $z[n] = x[n] * y[n]$ be the convolution of $x[n]$ and $y[n]$. Then (A) $z[0] = 0$ (B) $z[3] = 1 + \alpha + \alpha^2$ (C) $z[7] = \alpha^5 + \alpha^6 + \alpha^7$ (D) $z[10] = 1$ (E) None of the above.

10. (5%) Let $x[n]$ be a real and odd periodic signal with period $N = 7$ and Fourier series coefficients a_k. Given that

$$a_{15} = j, \; a_{16} = 2j, \text{ and } a_{17} = 3j,$$

    then (A) $a_0 = 0$ (B) $a_{-1} = j$ (C) $a_{-2} = -2j$ (D) $a_{-3} = 3j$ (E) None of the above.

11. (5%) The output $y(t)$ of a causal LTI system is related to the input $x(t)$ by the equation

$$\frac{dy(t)}{dt} + 10\,y(t) = \int_{-\infty}^{\infty} x(\tau) z(t-\tau)\,d\tau - x(t),$$

    where $z(t) = e^{-t}\,u(t) + 3\,\delta(t)$. If the impulse response $h(t)$ of the system is expressed as

$$h(t) = a\,e^{-t}\,u(t) + b\,e^{ct}\,u(t) + d,$$

    then (A) $a = \frac{1}{9}$ (B) $b = \frac{10}{9}$ (C) $c = -10$ (D) $d = 0$ (E) None of the above.

12. (5%) Consider the digital filter structure shown in Figure 1.

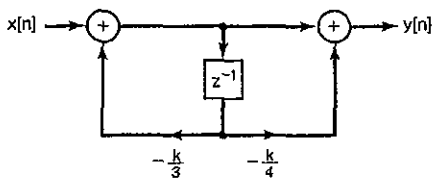

Figure 1: Digital filter

   Let the transfer function of the causal filter $H(z)$ be expressed as

$$H(z) = \frac{1 - a\,z^{-1}}{1 + b\,z^{-1}}.$$

   Then (A) $a = \frac{k}{3}$ (B) $b = \frac{k}{4}$ (C) The system is stable if and only if $\frac{|k|}{4} < 1$ (D) If $k = 1$, the response to $x[n] = \left(\frac{2}{3}\right)^n$ is $y[n] = \frac{5}{12}\left(\frac{2}{3}\right)^n$ (E) None of the above.

接次頁

※ 第二大題請於試卷內之「非選擇題作答區」標明題號依序作答。

第二大題－非選擇題：

13. *Suppose that there are two independent bandlimited AWGN channels available for communications.* The channels have the same bandwidth W but they have different noise power spectral densities $\frac{N_1}{2}$ and $\frac{N_2}{2}$, respectively. The total available transmit power is P. Denote the transmit power for the first channel as P_1 and the transmit power for the second channel as P_2. Note that $P_1 + P_2 = P$

    (a) (5%) Express the total capacity of the two channels as a function of N_1, N_2, W, P, P_1.

    (b) (5%) What is the optimal value of P_1 that maximizes the total capacity of the two channels? Express it as a function of N_1, N_2, W, P. (Note that the transmit power $P_1 \geq 0$.)

14. Consider a communication system operating under a non-Gaussian noise n, where the pdf of n is given by $p(n) = \frac{1}{\sigma\sqrt{2}} \exp\left(-\frac{|n|\sqrt{2}}{\sigma}\right)$. The transmit signal s is either A or $3A$, each with equal probability $\frac{1}{2}$. The corresponding receive signal is $r = s + n$.

    (a) (5%) What is the optimal decision rule for the receiver after receiving r? The decision rule has to be simplified to its simplest form. You also have to give rigorous explanation of why your answer is correct to get full credits.

    (b) (5%) What is the probability of error for the optimal receiver? Express it as a function of A and σ.

15. (10%) Consider a discrete-time signal $x[n]$ given by $x[n] = a^{|n|}$ with $|a| < 1$ and a continuous-time signal $y(t)$ given by

$$y(t) = \frac{1}{5 - 4\cos(2\pi t)}.$$

    (a) (5%) Find the discrete-time Fourier transform of $x[n]$.

    (b) (5%) Using the concept of duality, find the Fourier series coefficients of $y(t)$.

16. (10%) Given a signal $x(t)$, the integrated value of the signal over an interval T_0 is defined as

$$y(t) = \int_{t-T_0}^{t} x(\tau)\, d\tau.$$

    The integral $y(t)$ can also be obtained by transmitting $x(t)$ through a filter with transfer function given by

$$H(jw) = a\,\mathrm{sinc}(b)\, e^{(-jc)},$$

    Obtain the value of a, b, and c, and plot the magnitude response of the filter $|H(jw)|$.

試題隨卷繳回