1. (40 points) Let $\delta(t)$ be the Dirac delta function. Solve the following differential equations :

   (a)

   $$y''' + 4y'' + 5y' = \delta(t-3)$$

   with initial condition $y(0) = y'(0) = y''(0) = 0$.

   (b)

   $$y''(t) - \int_0^t \sin(t-\xi)y(\xi)d\xi = 0$$

   with initial condition $y(0) = 0, y'(0) = 1$.

2. (20 points) Find the general solution of the differential equation

   $$9x^2 f''(x) + 3xf'(x) + f(x) = 0$$

   for $x > 0$.

3. (20 points) Set

   $$A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \; X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}, \; f(t) = \begin{pmatrix} 0 \\ t \\ \cos t \end{pmatrix}. \tag{1}$$

   Find the solution to the differential system

   $$X'(t) = AX(t) + f(t) \tag{2}$$

   with the initial condition $x_1(0) = 1$, $x_2(0) = 0$ and $x_3(0) = 1$.

4. (20 points) Solve the following initial value problem

   $$x^{(4)}(t) - x^{(3)}(t) + x''(t) - x'(t) = \sin(t). \tag{3}$$

   with the initial condition $x(0) = x'(0) = x''(0) = x^{(3)}(0) = 0$ .

**試題隨卷繳回**