1. Find the general solutions of the differential equations:

$$y'' - 4y' + 5y = e^{2x} + e^x \sin(2x)$$

(10%)

2. Given,

$$\mathbf{B} = \begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

Find all matrices associated with the diagonal decomposition, $\mathbf{B} = \mathbf{PDP^{-1}}$, of $\mathbf{B}$, where $\mathbf{D}$ is the diagonal matrix formed from the eigenvalues of $\mathbf{B}$, and the columns of $\mathbf{P}$ are the corresponding eigenvectors of $\mathbf{B}$.

(10%)

3. (1) Can the method of Fourier Transform be used to solve the two-dimensional, steady-state problem with the conditions as indicated? Why?
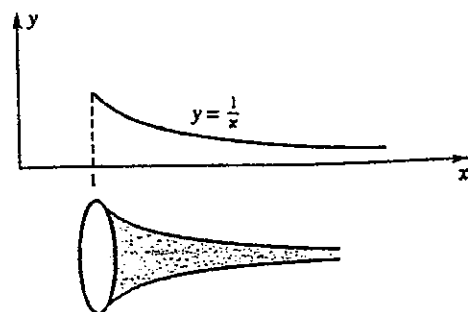
(10%)
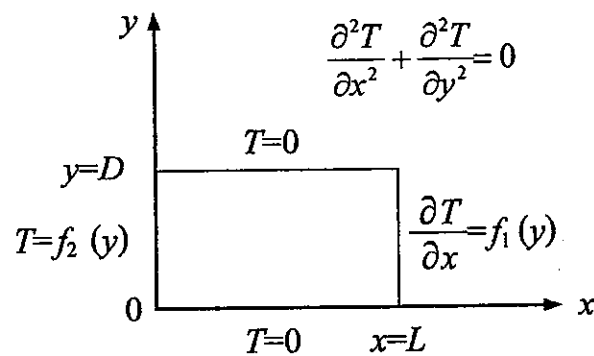
(2) Can the above problem (1) be solved by Laplace Transform? Why?

(10%)

4. For a Gabriel's horn (or Torricelli's Trumpet), Please find its volume (V) and curved surface area (S).

(10%)

見背面

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5. A Clairaut equation is of the form $y = xy' + g(y')$. Solve the special Clairaut equation $y = xy' + 1/y'$.

   (15%)

6. Given $f(x, y, z) = 3x^2 + 2y^2 - z^2 = 6$

   (15%)
   (1) find the tangent plane to the surface $f(x, y, z)$ at (2,1,3).
   (2) explain the meaning of $\nabla f$ and $\nabla f / |\nabla f|$

7. A tank contains 100 L of water in which 5 g of salt is dissolved. Forty liters of brine, each containing $(1 + \cos t)$ gram of dissolved salt, run into the tank per minute. The mixture, kept uniform by stirring, runs out at the same rate. Find the amount of salt $y(t)$ in the tank at any time $t$.

   (20%)

試題隨卷繳回