

第一大題：多重選擇題

*本大題各題答案應作答於答案卡，否則不予計分。

*每題有一個以上正確選項。每一選項答錯(應選而未選或不應選而選)扣 2 分。

*每題完整答對(無任何選項答錯)，該題得滿分 10 分。*每題未作答或答錯兩個以上選項者，該題以 0 分計。

1. Consider the following statements:

- i. The geometric distribution is memoryless.
- ii. The exponential distribution is memoryless..
- iii. The expectation of a random variable is always non-negative.
- iv. The value of a continuous probability density function $f_X(x)$ can not be larger than 1 at any x .

Which of the statements above is(are) TRUE?

(A) i (B) ii (C) iii (D) iv (E) None of the above.

2. Random variable X is Gaussian with mean = 0 and variance = 4. Consider the following statements:

- i. $E[|X|] = 0$.
- ii. $Var(|X|) = 5$.
- iii. $E[X^3] = 1$.
- iv. $E[(X - 3)^2] = 12$.

Which of the statements above is(are) TRUE?

(A) i (B) ii (C) iii (D) iv (E) None of the above.

3. Consider the following statements about two random variables X and Y :

- i. If X and Y are independent, then $Var(X + Y) = Var(X) + Var(Y)$.
- ii. If X and Y are uncorrelated, then $Var(X + Y) = Var(X) + Var(Y)$.
- iii. If X and Y are both zero-mean and X, Y uncorrelated, then $Var(X + Y) = Var(X) + Var(Y)$.
- iv. If X and Y are not independent, then $E[X + Y]$ may not equal to $E[X] + E[Y]$.

Which of the statements above is(are) TRUE?

(A) i (B) ii (C) iii (D) iv (E) None of the above.

4. Random variables X_1, X_2, X_3, X_4 are i.i.d. with the same distribution. X_i is exponentially distributed with mean = 1, $i = 1, 2, 3, 4$. Consider the following statements:

- i. The distribution of $W = X_1 + X_2 + X_3 + X_4$ is *Erlang*(4, 1/4).
- ii. $E[X_1 + X_2 + X_3 + X_4] = 4$.
- iii. $Var(X_1 + X_2 + X_3 + X_4) = 4$.
- iv. $E[e^{s(X_1+X_2+X_3+X_4)}] = \frac{1}{(s-1)^4}$.

Which of the statements above is(are) TRUE?

(A) i (B) ii (C) iii (D) iv (E) None of the above.

5. Random variables X_1, X_2 have the joint PDF:

$$f_{X_1, X_2}(x_1, x_2) = \begin{cases} 2 & 0 \leq x_2 \leq x_1 \leq C, \\ 0 & \text{otherwise,} \end{cases}$$

where C is a constant. Consider the following statements:

- i. The value of the constant C is $\frac{1}{2}$.
- ii. $E[X_1] = \frac{2}{3}$
- iii. $E[X_2^2] = \frac{1}{2}$.
- iv. $Cov(X_1, X_2) = \frac{1}{4}$.

Which of the statements above is(are) TRUE?

(A) i (B) ii (C) iii (D) iv (E) None of the above.

見背面

第二大題：非選擇題(本大題共 50 分，各題答案應作答於試卷，否則不予計分)

1. Find the general solution of the following differential equations: (25 scores)

(a) $\frac{dy(x)}{dx} + y(x) - e^{2x} = 0$ (8%)

(b) $\frac{dy(x)}{dx} - y(x) + e^{2x}y^2(x) = 0$ (7%)

(c) $y^{(4)}(x) - y^{(3)}(x) - 2y''(x) + 2e^x + 8 = 0$ (10%)

2. (a) Find the inverse Laplace transforms of (7.5%) (15 scores)

$$\frac{s^2}{s^2 - 2s + 3}$$

(b) Find the Laplace transform of (7.5%)

$$e^t \int_0^t e^{-2\tau} \sinh(t - \tau) \cos \tau d\tau$$

3. Solve the partial differential equation (10 scores)

$$\frac{\partial u}{\partial x^2} + 2 \frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} + u$$

試題隨卷繳回