國立臺灣大學 104 學年度碩士班招生考試試題

請詳細列出計算及推導過程，否則不予計分。題目前中括號 [ ] 內之數字為該題配分。

1. Define:

   (a) [5 points] Random experiment, probability set function and random variable.

   (b) [5 points] Moment generating function of a random variable.

   (c) [5 points] Characteristic function of a random variable.

   (d) [5 points] Unbiasedness, consistency, efficiency and sufficiency.

   (e) [5 points] Exponential family of distributions.

   (f) [5 points] Uniformly minimum-variance unbiased estimator.

   (g) [5 points] Likelihood function and maximum-likelihood estimator.

   (h) [5 points] Likelihood ratio test.

   (i) [5 points] Uniformly most powerful test.

   (j) [5 points] Linear statistical model and nonlinear statistical model.

2. [5 points] Let $X_1, \ldots, X_9$ be i.i.d. from the Normal distribution $N(\mu, \sigma^2)$ with unknown mean μ and variance σ^2. If we have three statistics, $Y_1 = \frac{1}{6}\sum_{i=1}^{6} X_i$, $Y_2 = \frac{1}{3}\sum_{i=7}^{9} X_i$, and $S^2 = \frac{1}{2}\sum_{j=7}^{9}(X_j - Y_2)^2$, please find the sampling distribution of the statistic $Z = \sqrt{2}(Y_1 - Y_2)/S$.

3. [10 points] Let $X_1, \ldots, X_n$ be i.i.d. from the following continuous distribution:

$$f_\theta(x) = \theta(1+x)^{-(1+\theta)}, \quad x > 0;$$

   with a unknown parameter $\theta > 0$.

   (a) Find an estimator of θ using the method of moments.

   (b) Find the Maximum Likelihood Estimate (MLE) of θ.

4. [15 points] Let $X_1, \ldots, X_n$ be i.i.d. from the distribution with pdf

$$f(x;\theta) = \frac{3x^2}{\theta} e^{-x^3/\theta}, \quad x > 0.$$

   (a) Find the sampling distribution of $\sum_{i=1}^{n} X_i^3$.

   (b) Find the UMP (uniformly most powerful) test of size α for testing $H_0 : \theta = \theta_0$ versus $H_a : \theta > \theta_0$.

5. [20 points] Let $X_1, \ldots, X_n$ be i.i.d. having the Poisson(θ) distribution. We want to estimate $g(\theta) = e^{-\theta}$.

   (a) Prove that $S \equiv \sum_{i=1}^{n} X_i$ is sufficient and complete for θ.

   (b) An unbiased estimator is of the form, $T(X_1, \ldots, X_n) = \mathrm{I}_{[X_1=0]}$. Please find the UMVUE (uniformly minimum variance unbiased estimator) for $g(\theta)$.

   (c) Find the Cramér-Rao Lower Bound for the variance of unbiased estimator of $g(\theta)$. Decide whether the UMVUE of $g(\theta)$ reaches the Cramér-Rao Lower Bound.

試題隨卷繳回