國立臺灣大學 104 學年度碩士班招生考試試題

1. Let p and q be positive integers, q odd, and $f(x) = x^{p/q}$. Specify conditions on p and q so that (a) f has a vertical tangent at (0, 0), (b) f has a vertical cusp at (0, 0). (10%)

2. Sketch the graph of $f(x) = \frac{x^2-3}{x^3}$, and indicate the extreme values, inflection points, concavity, and asymptotes (if any). (20%)

3. Calculate $\int \frac{e^t}{e2^t+5e^t+6} dt$. (10%)

4. Determine the volume of the solid generated by revolving the cardioid $r = (1-\cos\theta)$ about the x-axis. (10%)

5. Let a and b be positive. Find $\lim_{x\to\infty}\left[\left(a^{1/x}+b^{1/x}\right)/2\right]^x$. (10%)

6. Determine whether the series $\sum_{k=2}^{\infty} a_k$ converges or diverges. If it converges, find the sum. $a_k = \sum_{n=2}^{\infty}\left(\frac{1}{k}\right)^n$. (10%)

7. Find the length of the curve $\vec{r}(t) = cost\vec{i} + sint\vec{j} + coshtk$ from t = 0 to ln2. (10%)

8. Maximize $2x + 3y + 5z$ on the sphere $x^2+y^2+z^2 = 19$. (10%)

9. Take Ω as the parallelogram bounded by $x-y=0$, $x-y=\pi$, $x+2y=0$, $x+2y=\pi$. Evaluate $\iint_{\Omega} sin3x\,dx\,dy$. (10%)

試題隨卷繳回