國立臺灣大學 104 學年度碩士班招生考試試題

(50%) Problem 1. Measuring liquid viscosity using the falling cylinder device.

Consider the falling cylinder viscometer device as shown in **Fig. 1** (next page). The device consists of a very long vertical cylindrical tube with an inner radius of R, and the cylindrical cavity inside the tube is filled with an incompressible viscous liquid of constant viscosity η and density ρ. Immersed in the liquid is a solid cylindrical plug having a radius of αR ($\alpha < 1$), a length of L, and a density of ρ_p. Under the influence of gravitation, i.e., $\bar{g} = -g\bar{i}_z$, a typical viscosity measurement is made by allowing the plug to fall or descend slowly through the viscous liquid and measuring the steady terminal descent velocity, U, of the cylindrical plug. Assuming that the centerline axis of the cylindrical plug coincides with that of the cylindrical tube, i.e., the gap distance between the tube and the plug, $R - \alpha R$, is maintained constant, and that the liquid flow within gap between the tube and the plug is *slow lubricating flow*, please show by step-by-step derivation that the viscosity of the liquid can be expressed as

$$\eta = \frac{g(\rho - \rho_p)\alpha^2 R^2}{2U}\left[\ln(\alpha) + \left(\frac{1-\alpha^2}{1+\alpha^2}\right)\right].$$

You may need the continuity and Navier-Stokes equations in cylindrical coordinates, i.e.,

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_r) + \frac{1}{r}\frac{\partial u_\phi}{\partial \phi} + \frac{\partial u_z}{\partial z} = 0,$$

$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\phi}{r}\frac{\partial u_r}{\partial \phi} - \frac{u_\phi^2}{r} + u_z\frac{\partial u_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \eta\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru_r)\right) + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \phi^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial u_\phi}{\partial \phi}\right] + \rho g_r,$$

$$\rho\left(\frac{\partial u_\phi}{\partial t} + u_r\frac{\partial u_\phi}{\partial r} + \frac{u_\phi}{r}\frac{\partial u_\phi}{\partial \phi} + \frac{u_r u_\phi}{r} + u_z\frac{\partial u_\phi}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \phi} + \eta\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru_\phi)\right) + \frac{1}{r^2}\frac{\partial^2 u_\phi}{\partial \phi^2} + \frac{\partial^2 u_\phi}{\partial z^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \phi}\right] + \rho g_\phi,$$

and

$$\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\phi}{r}\frac{\partial u_z}{\partial \phi} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \eta\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \phi^2} + \frac{\partial^2 u_z}{\partial z^2}\right] + \rho g_z.$$

You are free to make any further reasonable and educated assumptions to help solving this problem.

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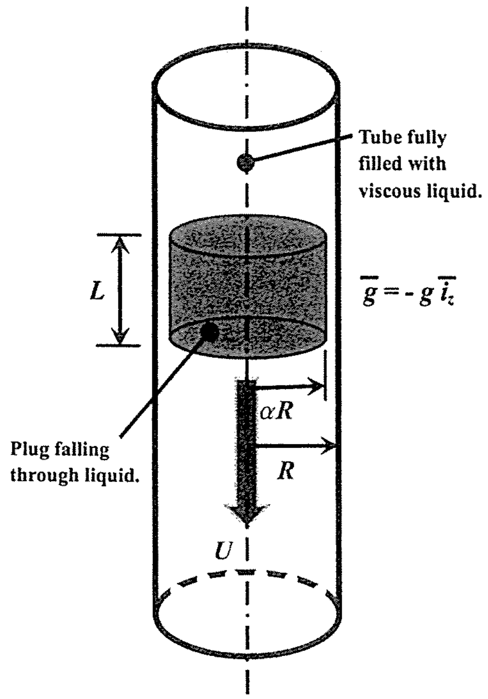

Fig. 1. The falling cylinder viscometer (Problem 1).

(50%) Problem 2. A two-dimensional discharge problem.

Water of density ρ and viscosity μ is being discharged from kettle with a long thin feeding tube of length L and inner diameter D as sketched in Fig2 (left). **Treat this as a two-dimensional problem** and the kettle base is A.

(1) 24% Estimate the time T to evacuate water of volume V. Assume the flow developed in the tube remains laminar throughout the discharge and both the entrance effect and transient effect are negligible.

(2) 16% To speed up the discharge, Student A proposes to tilt the kettle to an angle θ while Student B suggests to cut the feeding tube short. Explain why each method will or will not work.

接次頁

國立臺灣大學 104 學年度碩士班招生考試試題

(3) 5% What phenomenon may occur if the flow within the tube becomes turbulent after speed-up?

(4) 5% What phenomenon may occur if separation takes place?

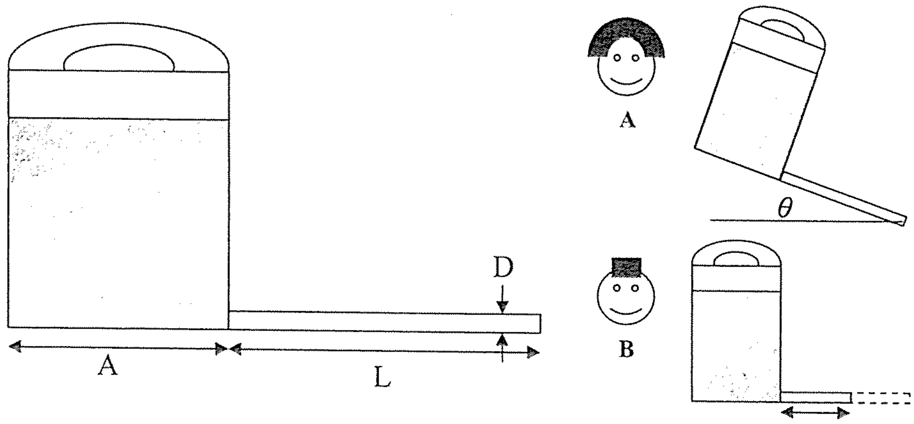

Fig. 2.

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