1. Assume a (real valued) function \( y = f(x) \) satisfy \( y^2 + 9y = x^3 + x \). Suppose \( g(x) \) is a (real valued) function satisfying \( f(g(x)) = x \).
   (a) (5%) The value \( g(1) \) must be 2. True or false? Answer: _ (1)_.
   (b) (10%) The derivative \( g'(1) = _ (2)_.

2. An experiment detects that a particle at \((1, 0)\) (on the \(xy\)-plane) is moving towards the north at the speed 3 meters per minute. At the same time another particle at \((-2, 1)\) is moving towards the east at the speed 4 meters per minute.
   (a) (5%) Let \( s(t) \) be the distance (in meters) between the two particles in \( t \) minutes. \( s(t) = _ (3)_.
   (b) (10%). Suppose at \( t = t_0 \) the two particles are closest to each other. Then \( t_0 = _ (4)_.

3. (10%) Compute the indefinite integral \( \int \frac{dx}{(x-1)(x+1)} = _ (5)_.

4. (10%) Let \( \Omega \) be a region (on the \(xy\)-plane) enclosed by \( x = \sqrt{\ln y}, x = 0 \) and \( y = e \). Let \( S \) be the solid obtained by revolving \( \Omega \) about the \(y\)-axis. Let \( V \) be the volume of \( S \). \( V = _ (6)_.

5. Let \( f(x, y) = x^2 - e^{xy} \) and the surface \( S \) be the graph of the function \( z = f(x, y) \). Let \( P = (1, 0, 0) \in S \) and \( p = (1, 0) \) in the \(xy\)-plane.
   (a) (10%) If the unit vector \( u \) (in the \(xy\)-plane) at \( p \) is the direction (among all directions at \( p \) along which the height (i.e. the value of \( z \)) of \( S \) increases most rapidly, then \( u = _ (7)_.
   (b) (10%) Write \( H \) for the plane \( x + 2y + 3z = 1 \) and the curve \( C \) for the intersection \( S \cap H \). Let \( L \) be the tangent line to \( C \) at \( P \) and \( N \) be the plane perpendicular to \( L \) at \( P \). Then the equation of \( N \) is _ (8)_.

6. Let \( R \) be the region enclosed by \( y = x, y = x - 2, y = 1 \) and \( y = 0 \).
   (a) (10%) Let \( A \) be the area of \( R \). \( A = _ (9)_.
   (b) (5%) The double integral \( \iint_R \sqrt{x - y} \, dxdy = _ (10)_.

7. (15%) Let \( \Omega \) be the region \( \{(x, y) \mid (x-1)^2 + y^2 < 1 \} \). Evaluate the double integral \( \iiint_\Omega \frac{1}{x} \, dxdy \).

試題隨卷繳回