I. Suppose that $f(x)$ is a function satisfying

$$e^x \le f(x) \le e^x + x^2, \quad |x| < 1.$$

Find

(a) $f(0)$. (5%)

(b) $f'(0)$. (10%)

II. Suppose that f is a continuous function satisfying

$$f(x) = 2 + \int_0^x \frac{f(t)}{(t+2)(t+3)} dt, \quad x \ge 0.$$

Find

(a) $f(0)$. (5%)

(b) $f(3)$. (10%)

III. Compute the following :

(a) $\lim_{n\to\infty} a_n$. Here $a_1 = \sqrt{6}$, $a_2 = \sqrt{6+\sqrt{6}}, a_3 = \sqrt{6+\sqrt{6+\sqrt{6}}}, \ldots$ (5%)

(b) $\lim_{t\to 0}(\int_0^1 (1+x)^t dx)^{\frac{1}{t}}$. (10%)

IV. (15%) Find all absolutely extreme values of the function

$$f(x,y) = x^2 + 3y^2 + y$$

subject to the constraint

$$x^2 + y^2 \le 1.$$

V. (10%) Find the normal line for the surface

$$z = x^2 - xy - y^2$$

at the point $(1, 1, -1)$.

VI. Evaluate the following :

(a) $\iint_R e^{x^2+y^2} dxdy$. Here $R = \{(x,y) : x^2 + y^2 \le 1, \ y \ge 0\}$. (10%)

(b) $\int_0^\infty e^{-x^2} dx$. (5%)

VII. Find the area of the region enclosed by the following :

(a) (5%)

$$r = 2a\cos\theta, \quad a > 0.$$

(b) (10%)

$$r = 2(1 + \cos\theta).$$

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