1. Considering two species, snowshoe hare and Canada lynx, and the lynxes prey on the hares. Hares have unlimited food supply and to reproduce exponentially, unless subject to predation. The growth rate of the hare is proportional to the population of the hare with the growth rate coefficient α. The rate of predation upon the hares is assumed to be proportional to the rate at which the lynxes and the hares meet. It is proportional to the product of the lynx and hare populations with a positive coefficient β. The growth rate of the lynx is proportional to the predation with a positive coefficient γ. The death rate of the lynx is proportional to its population with the death rate coefficient ε.
   (a) Write down the system of different equations for the population of lynx and hare using the assumptions above. (5%)
   (b) For $\alpha = 3, \beta = 2, \gamma = 2, \varepsilon = 1$, determine the type and stability of each critical point (10 %)
   (c) With initial hare population of k, find the population of the hare versus time when the initial lynx population is zero. (5%)
2. Solve the non-homogeneous ODE (10%)
   $x^2y'' + xy' + 9y = x^3$
3. Solve the non-homogeneous ODE (20%),
   $y'' + 0.4y' + 4.04y = r(t)$,

   $r(t) = 2\cosh t$, if $0 < x < 1$ and 0 if $t > 1$;

   $y(0) = 1, y'(0) = -1$ (Note: $\mathcal{L}\{\cosh at\} = \frac{s}{s^2-a^2}$)
4. Each year 1/10 of the people outside the capital city move in, and 2/10 of the people inside the capital city move out. This is a Markov process which shows the probability of moving to the next state depends only on the present state.
   (a) Formulate the transition probability equations of this Markov chain. (10%)
   (b) Calculate the steady state of the population distribution. (10%)
   (c) Plot the eigenvectors, and steady state of transition probability matrix on the figure. (10%)
5. Solve the two-dimensional wave equation in polar coordinates. Using, the method of separation of variables, determine solutions that are radially symmetric. (20%)

   $\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right)$

   $u(R,t) = 0$ for all $t \geq 0$

   $u(r,0) = f(r),\ u_t(r,0) = g(r)$