Problem 1 (15 points) Consider the following linear programming problem:

$$\begin{aligned}&\text{Max } 2x_1+4x_2+3x_3\\&\text{subject to}\\&\quad x_1+3x_2+2x_3\le 30\\&\quad x_1+x_2+x_3\le 20\\&\quad 3x_1+5x_2+3x_3\le 75\\&\quad x_1,x_2,x_3\ge 0.\end{aligned}$$

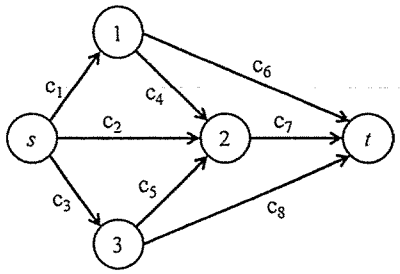
Assume that in the optimal solution $x_1>0$, $x_2=0$, and $x_3>0$.

(a) (*5 points*) Use the given information and describe how you adapt the simplex method to solve the problem in the minimum possible number of iterations.

(b) (*5 points*) Solve the problem based on the procedure developed in (a).

(c) (*5 points*) Based on the optimal solution from (b), what are $\boldsymbol{B}$ (basis matrix) and $\boldsymbol{B}^{-1}$?

Problem 2 (15 points) Consider the following network with 5 nodes and 8 directed arcs where c_i denotes the capacity of arc i (i = 1,…, 8).



(a) (*5 points*) List all possible paths from node s to node t.

(b) (*10 points*) Let f_j be the flow through path j listed in (a). Formulate a programming model that maximizes the flow from node s to node t.

Problem 3 (20 points) Consider the following problem:

$$\begin{aligned}&\text{Max } 3x_1-x_1^2+4x_2^2-x_2^3\\&\text{subject to}\\&\quad x_1+x_2\le 3\\&\quad x_1 \text{ and } x_2 \text{ are} \ge 0 \text{ and integers.}\end{aligned}$$

This problem can be reformulated in two ways as an equivalent pure Binary Integer Programming (BIP) problem, depending on the definitions of the binary variables.

(a) (*10 points*) Formulate a BIP problem where the binary variables are interpreted as: $y_{ij}=1$ if $x_i=j$ (i = 1, 2, and j = 1, 2, 3), and $y_{ij}=0$ otherwise.

(b) (*10 points*) Formulate a BIP problem where the binary variables are interpreted as: $y_{ij}=1$ if $x_i\ge j$ (i = 1, 2, and j = 1, 2, 3), and $y_{ij}=0$ otherwise.

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Problem 4 (9 points) Consider a bank with two clerks. Three customers, ***A***, ***B***, and ***C***, enter the bank simultaneously. ***A*** and ***B*** go directly to the clerks, and ***C*** waits until either ***A*** or ***B*** leaves before he begins service. What is the probability that ***B*** is still in the bank after the other two have left when

(a) (*3 points*) the service time is exactly five minutes?

(b) (*3 points*) the service time is equally likely to be 2, 4, or 6 minutes?

(c) (*3 points*) the service time is exponentially distributed with mean 5 minutes?

Problem 5 (11 points) EMS is an insurance company with a long history. EMS charges a customer according to his or her accident history.

- A customer who has had no accident during the last two years is charged \$500 / year.
- A customer who has had an accident during each of the last two years is charged \$2,000 / year.
- A customer who has had an accident during only one of the last two years is charged \$1,200 / year.
- ✧ A customer who has had an accident during the last year has a 20% probability of having an accident during the current year.
- ✧ For a customer who had no accident during the last year, there is only a 5% probability that he or she will have an accident during the current year.

(a) What is the average premium paid by a long-term customer of EMS?

Problem 6 (15 points) IIE is a company that sells tablet computers to NTU students.

- IIE is equally likely to sell 200 or 400 tablets on each day.
- Each tablet is sold for \$80.
- IIE orders tablets from a supplier every morning, and orders are delivered immediately. When IIE orders tablets from the supplier, IIE pays \$5000 for order processing plus \$50 for each tablet. (i.e. If IIE orders x tablets from the supplier, the total cost is $5000 + 50x$).
- A holding cost of \$10 per tablet is assessed against each unsold tablet at the end of a day.
- IIE can store 400 tablets at most after each day.
- The number of tablets ordered by IIE must be a multiple of 100.
- IIE has no inventory on hand at the beginning of this week.

(a) Assume that each unsold tablet has a value of \$60 at the end of the third day. Determine an ordering policy that maximizes the expected profit earned during the first three days of this week. (profit = revenue – costs)

Problem 7 (15 points) Taiwan Airlines faces demand during both high and low seasons. For Taiwan Airlines, a year consists of 50% high season dates and 50% low season dates.

- If a price of x dollars is charged during the high season, daily customer demands are $60 - 0.5x$ units.
- If a price of y dollars is charged during the low season, daily customer demands are $40 - y$ units.
- The company must have sufficient capacity to meet demands during both the high and the low seasons.
- Capacity remains unchanged over the year, and it costs \$10 per day to maintain each unit of capacity.

(a) Determine how Taiwan Airlines can maximize its profit.

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