1. (15%)

Optimization is a branch of science in which humans had an abiding interest since prehistoric times. Nowadays all the decisions that we make at work, and those affecting our personal lives, usually have the goal of optimizing some desirable characteristics. Product mix problems are an extremely important class of problems that manufacturing or service firms face. Normally the company can make a variety of products using the raw materials, machinery, labor force, and other resources available to them. The problem is to decide how much of each product to produce in a period, to maximize the total profit subject to the availability of needed resources. To model this, we need data on the units of each resource necessary to produce one unit of each product, any bounds (lower, upper, or both) on the amount of each product produced per period, any bounds on the amount of each resource available per period, and the cost or net profit per unit of each product produced. Assembling this type of reliable data is one of the most difficult jobs in constructing a product mix model for a firm, but it is very worthwhile. A product mix model can be used to derive extremely useful planning information for the firm. Suppose The GoodNTU Bakery bakes two types of bread mixes A and B using two raw materials R_1 and R_2. The following table gives the necessary data.

Raw material	Units needed to make 1 unit of		Units available
	A	B	
R_1	1	2	6000
R_2	2	1	8000
Net profit per unit made	7	5	
Maximum demand	3500	2500	

(a) Formulate the problem of determining how many units of A and B to make, as a linear programming model. (3%)

(b) Solve the problem geometrically. (4%)

(c) Determine the shadow prices associated with all the right-hand-side constants in the model. (4%)

(d) How much extra profit can the bakery make if the supply of R_1, R_2 is increased by one unit? (4%)

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2. (23%)

Consider the linear program

$$\begin{array}{llllllll} \text{minimize} & -4x_1 & - & 5x_2 & + & 3x_3 & & \\ \text{subject to} & x_1 & + & x_2 & + & x_3 & = & 10 \\ & x_1 & - & x_2 & & & \ge & 1 \\ & x_1 & + & 3x_2 & + & x_3 & \le & 14 \\ & x_1, & & x_2, & & x_3 & \ge & 0 \end{array}$$

Let x_4 be the surplus variable for the second constraint and let x_5 be the slack variable for the third constraint. The unique optimal solution is (x_1^*, x_2^*, x_3^*)=(8, 2, 0). The dual problem is given by

$$\begin{array}{llllllll} \text{maximize} & 10y_1 & + & y_2 & + & 14y_3 & & \\ \text{subject to} & y_1 & + & y_2 & + & y_3 & \le & -4 \\ & y_1 & - & y_2 & + & 3y_3 & \le & -5 \\ & y_1 & & & + & y_3 & \le & 3 \\ & y_1 & & & & & & \text{urs}^* \\ & & & y_2 & & & \ge & 0 \\ & & & & & y_3 & \le & 0 \end{array}$$

* indicates "unrestricted sign."

Let y_4, y_5, y_6 be the slack variables for the first, second, and third constraints, respectively, in the dual program.

(a) Write the optimal primal basic feasible solution, i.e. what is $(x_1^*, x_2^*, x_3^*, x_4^*, x_5^*)$? (5%)

(b) Use complementary slackness to find the optimal dual basic feasible solution, i.e. what is $(y_1^*, y_2^*, y_3^*, y_4^*, y_5^*, y_6^*)$? (6%)

(c) For the basic vector $x_B = \begin{bmatrix} x_1 \\ x_2 \\ x_4 \end{bmatrix}$, the optimal basis inverse is given as

$$B^{-1} = \begin{bmatrix} \frac{3}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ 2 & -1 & -1 \end{bmatrix}.$$

For what range of right-hand-side values will the shadow price of the first constraint be valid? For what range of values of the first coefficient of the objective function, c_1, will the optimal solution point remain unchanged? (12%)

3. (12%)

Consider the following unconstrained nonlinear program with two variables.

$$\text{minimize } \pi(x_1, x_2) = -2x_1^2 - x_2^2 + x_1x_2 - 10x_1 + 6x_2$$

(a) What is the Hessian matrix of this problem? Is this positive semi-definite? (6%)

(b) Solve this nonlinear program. (6%)

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4. (8%)

Suppose that each child in a family is equally likely (and independently) to be a boy or a girl. Now, a family is randomly chosen from Taipei City. This family has three children, and at least one of them is a boy.

(a) What is the probability that the family has exactly two boys?

5. (27%)

The Department of Health is planning to allocate four medical teams to three different cities to improve health education. Each medical team can be allocated to only one city. The decision maker needs to determine how many medical teams (if any) to allocate to each of the three cities to maximize the total effectiveness. The total effectiveness is measured by the sum of additional person-year of life in all three cities.

The following table lists estimated additional person-year of life if *x* medical teams are allocated to City 1, 2, and 3.

# of teams (*x*)	City 1	City 2	City 3
0	0	0	0
1	50	20	45
2	70	45	70
3	80	75	90
4	100	110	105

(For example, if three medical teams are allocated to City 1, the public health in City 1 will be improved by 80 additional person-year of life.)

(a) Formulate this decision problem using dynamic programming. How many medical teams should be allocated to each of the three cities to maximize the total effectiveness? (18%)

(b) What is the maximum total effectiveness under optimal medical team allocation? (9%)

6. (15%)

Trials are performed in sequence. If the last two trials were successes, then the next trial is a success with probability 0.8; otherwise the next trial is a success with probability 0.5.

(a) In the long run, what proportion of trials are successes?

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