※ 注意：請於試卷內之「非選擇題作答區」標明題號依序作答。

Instructions:

- This exam has two parts.
  Part I consists of fill-in-the-blank problems. Only the clearly labeled answers will be graded.
  Part II consists of partial credit problems. Any answer without explanation will not receive credit.
- No electronic devices or computer algebra systems allowed for this exam.
- Usage of any theorem/formula must be clearly stated.

Part I: 4 points for each blank.

(1) Evaluate $\lim_{x\to 0^+}\dfrac{\sqrt{x}-\sqrt{\sin x}}{\sqrt{x^7+x^5}} = \underline{\ (1)\ }$.

(2) Let $y=f(x)$ be a function defined implicitly by the equation $2(x^2+y^2)^2=25(x^2-y^2)$ near $x=3, y=1$. By linear approximation $f(2.87)\approx \underline{\ (2)\ }$.

(3) Let $\mathcal{R}$ be the region described by $\{(x,y)\mid \cos x\le y\le \sec^2 x,\ 0\le x\le \frac{\pi}{4}\}$. The volume obtained by rotating $\mathcal{R}$ about the y-axis is $\underline{\ (3)\ }$.

(4) The area of the region inside the polar curve $r=5+3\cos\theta$ is $\underline{\ (4)\ }$.

(5) The 4th nonzero term of the Maclaurin series of the function $f(x)=\sqrt{9+x^2}$ is $\underline{\ (5)\ }$.

(6) The coefficient of x^{2023} in the Maclaurin series of $g(x)=x^4\tan^{-1}(4x^3)$ is $\underline{\ (6)\ }$.

(7) The tangent plane of the surface $xy^2z^3=8$ at the point $(2,2,1)$ is given by the equation $\underline{\ (7)\ }=0$.

(8) Evaluate $\int_0^4\int_{\sqrt{x}}^2 x\cos(y^5)\,dy\,dx = \underline{\ (8)\ }$.

(9) Evaluate $\int_0^2\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{-x^2-y^2}\,dx\,dy = \underline{\ (9)\ }$.

(10) Let E be a solid in the first octant. The largest possible value of $\iiint_E (9-x^2-y^2-z^2)\,dV$ is $\underline{\ (10)\ }$.

Part II: 15 points for each problem.

(11) Sketch the curve $y=\left(x^{-1/2}\ln x\right)$ and its asymptotes. Find the intervals of increase/decrease and concavity. Label local extrema and inflection points if any.

(12) Evaluate the definite integral $\int_1^2 x\ln(x^2-4x+5)\,dx$.

(13) A logistic population model with relative growth rate 0.1 per year and carrying capacity 50 thousand can be expressed by the differential equation $\dfrac{dP}{dt}=0.1P\left(1-\dfrac{P}{20}\right)$, with P in thousands and t in years. Given that the initial population is 9 thousand. Find the population size after 20 years. (If you memorized the formula, then you need to derive it for this problem.)

(14) Find the extreme values of $f(x,y,z)=z$ subject to the constraints $x^2+y^2=z^2$ and $x+2y+4z=16$.

試題隨卷繳回