（共 2 頁，可攜帶計算機做答）

一、填充題(60%，每格 3%，請依序填寫答案，編號、數值與單位須標示清楚；

[g = 9.8 m/s^2, gas constant R = 8.31 J/mol·K, electrostatic constant $k = 1/(4\pi\varepsilon_0) = 8.99\times10^9$ N·m^2/C^2; permeability constant $\mu_0 = 4\pi\times10^{-7}$ H/m, or $4\pi\times10^{-7}$ T·m/A; charge $e = 1.6\times10^{-19}$ C, Planck constant $h = 6.63\times10^{-34}$ J·s, speed of light $c = 3.0\times10^8$ m/s = $1/\sqrt{\varepsilon_0\mu_0}$]

1. A fully fueled rocket has a mass of 21,000 kg, of which 15,000 kg is fuel. The burned fuel is spewed out the rear at a rate R = 150 kg/s with a speed of 3000 m/s relative to the rocket. If the rocket is fired vertically upward, calculate: (a) the thrust of the rocket (1)____, and (b) its final velocity at burnout (fuel is fully consumed): (2)____. Ignore the air resistance. [*Hint*: 1st rocket eq.: $Rv_{rel} = Ma$; 2nd rocket eq.: $v_f - v_i = v_{rel}\ln(M_i/M_f) - gt$, where g is considered as a constant value and t is the time for fuel burnout.].

2. Figure 1 shows a reversible cycle through which 0.2 mol of a monatomic ideal gas is taken. Volume $V_d = 4.0V_a$, $V_b = V_a$, and $V_c = V_d$. Process bc is an isothermal expansion, with p_b = 10.0 atm (=1.013×10^6 Pa) and $V_b = 1.0\times10^{-3}$ m^3. Process ***da*** is adiabatic (pV^γ = constant with $\gamma \equiv C_p/C_V$) with p_a = 5.0 atm. For the cycle, find (a) the temperature at point ***a***, T_a: (3)____, (b) the energy added to the gas as heat, (4)____ (c) the efficiency of the cycle, (5)____, and (d) the entropy change ΔS during process ***bc***, (6)____.

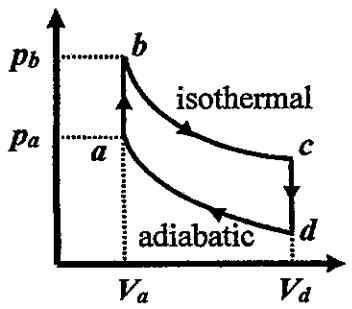

Fig.1

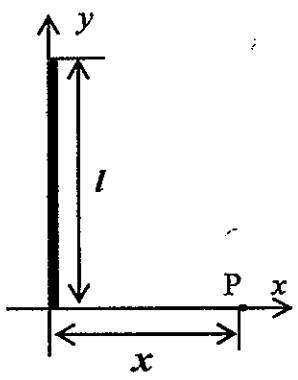

Fig. 2

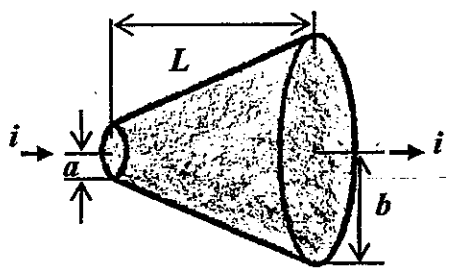

Fig. 3

3. A thin rod of length l lines on the y axis with one end on the origin, carrying a non-uniform line density given by $\lambda=\lambda_o y/l$, (a)Calculate the potential at point P (See Figure 2) on the x axis, with a distance x from the origin. (7)____ (b) Integrate the charge density to find the total charge on the rod. (8)____ (c) Find the result that this expression reduce to an expected result when $x >> l$. (9)____(d) Find the magnitude of field component E_x at P: (10)____. [Note that $(1+x)^n \approx 1 + nx$, for $x << 1$.]
4. In Figure 3, current is set up through a truncated right circular cone of resistivity ρ, left radius a, right radius b, and length L. Assume that the current density is uniform across any cross section taken perpendicular to the length. (a)What is the resistance of this cone? (11)____ (represented with ρ, a, b, and L.) (b) This cone was made from copper material. Taking $\rho = 1.69\times10^{-8}$ Ω·m for copper at 293 K, left radius a= 0.20 mm, right radius b= 0.60 mm, and length L = 10.0 cm, calculate the resistance of the cone. (12)____ (c) Taking the temperature coefficient of resistivity, $\alpha = 4.3\times10^{-3}$ K^{-1} for copper, re-estimate the resistance at 400 K. (13)____. [*Hint*: $\rho = \rho_0(1+\alpha\cdot\Delta T)$, where ΔT is the change of temperature.]

5. (a) Derive the formula for the magnetic field of an ideal long air-core solenoid carrying a current I (14)____(represented with I, the number of turns per unit length of this solenoid n, and permeability constant μ_0.). (b) Find the magnitude of the magnetic field at the center of the endpoint of a long solenoid (length $l >>$ radius R). (also expressed with n, I, μ_0.) (15) (c) If the current of the above-mentioned long solenoid increases at a rate of 200 A/s, and the number of coil turns per unit length n is 1000 turns; the radius of the solenoid R is 3.0 cm, then an aluminum alloy with a radius of 5.0 cm and a resistance of 0.0001 Ω is placed at the endpoint of this long air-core solenoid as shown in Figure 4. Approximately estimate the induced current in amperes? (16)____.

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6. A 20-mW helium-neon laser (λ = 632.8 nm) emits a beam of circular cross-section whose diameter is 1 mm. Assume the beam has a uniform intensity over its cross section. (a) Find the energy of one photon for this laser in unit of eV:__(17)__ (b) Find the Poynting vector of this laser beam.__(18)__ (c) What is the intensity of this laser beam?__(19)__ (d) What total momentum per unit length of the beam is carried?__(20)__

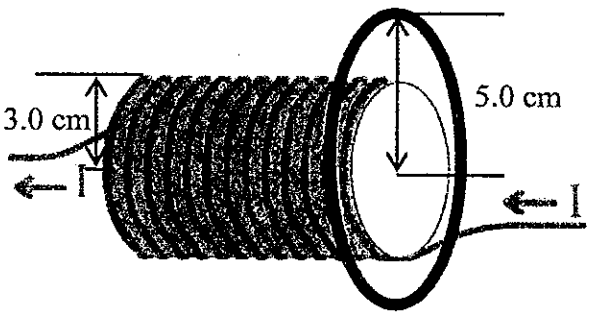

Fig. 4

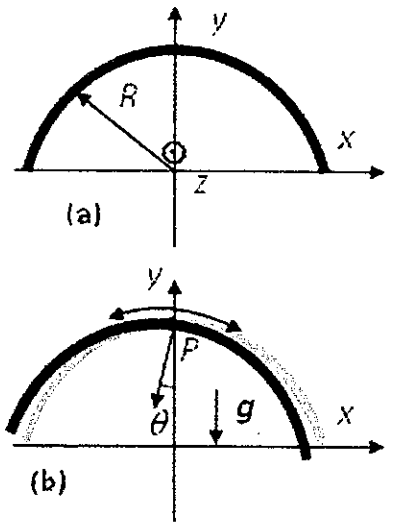

Fig. 5

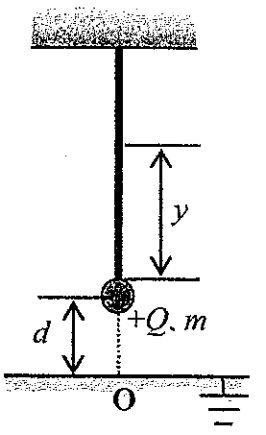

Fig. 6

二、計算題 (40%，計算過程之列式須清楚，為給分之依據)

1. A homogeneous thin half circle of radius R with a total mass M is symmetric about the y axis. as shown in Figure 5(a). (a) Find the coordinate of the center of mass (COM), (x_{COM}, y_{COM}). (b) Find the rotation inertia when it rotates around the axis passing through its COM (in parallel with the z-axis). (c) If the half circle is rotated in the xy plane with a small angle θ on a frictionless pivot at point P which is on the y axis, as shown in Fig. 5(b), and released. Show that the half circle will execute simple harmonic motion, and find the period T of simple harmonic motion, where the direction of gravitational acceleration $\boldsymbol{g}$ is in $-\hat{y}$. (15%)

2. A uniform insulating rope of mass M and length L hangs from a ceiling. A small metal ball with charge $+Q$ and mass m is suspended at the bottom of the rope, which is above an infinite grounded plane at a distance d from it, as shown in Figure 6. There are surface charges induced in the grounded plane. (a) Find the electric force acting on the charged metal ball. (b) The surface charge density at point O, which is directly below the ball. (c) Find the speed of a transverse wave on the rope due to the electric and gravitational forces, which is a function of y, the distance from the lower end. (15%) [*Hint*: the speed of a transverse wave on a string $v = \sqrt{\tau/\mu}$ with tension τ and linear mass density μ.]

3. Try to derive the wave equation of the electromagnetic wave from Maxwell's equations, and obtain that its propagation speed is the speed of light c in vacuum. (10%)

試題隨卷繳回