題號:17

國立臺灣大學112學年度轉學生招生考試試題

科目:微積分(B)

共 2. 頁之第 1 頁

• 注意: 請於試卷上『非選擇題作答區』標明題號作答。

- 禁止使用計算機或類似功能電子器具。
- PART I. Fill in the blanks. Each blank is worth 7 points. Only the final clearly labeled answer will be graded.
 - 1. Evaluate

$$\lim_{x \to 0} \left(\frac{\sqrt{x+4} - 2}{\tan^{-1}(\pi x)} \right) = \underline{\qquad (1)}$$

- 2. The curve described by $x \sqrt{xy} + y = 3$ passes through the point (4,1). Find an equation for the tangent line to the curve at (4,1). (2)
- 3. The absolute maximum value of the function $f(x) = x^6 e^{-2x^2}$ is ______(3)
- 4. The graph of the function $g(x) = \frac{\ln(x^3)}{\sqrt{x}}$ has an inflection point at $x = \underline{\hspace{1cm}}$ (4)
- 5. Solve for the function f that satisfies

$$\int_{\sqrt{x}}^{4} \frac{f(t^2)}{\ln t} dt = e^{(x-16)^2} - \frac{x}{16} , \quad x > 1.$$

$$f(x) = \underline{\qquad \qquad (5) \qquad \qquad }.$$

6. The volume generated by rotating the region under the curve $y = \frac{1}{\sqrt{x}(\sqrt{x}+1)}$ from x = 1 to x = 4 about the x-axis is ______.

見背面

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共 2 頁之第 2 頁

- 7. Determine if the improper integral $\int_2^\infty \frac{dx}{x(x^2-1)^{3/2}}$ is convergent or divergent. Evaluate the improper integral if it is convergent. (7)
- 8. Find the point on the surface given by $\sqrt{x} + \sqrt{8y} + \sqrt{27z} = 14$ that is closest to the origin.
- 9. Evaluate

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{2-\sqrt{4-x^2-y^2}}^{2+\sqrt{4-x^2-y^2}} \sqrt{x^2+y^2+z^2} \ dz \ dy \ dx = \underline{\qquad (9)}$$

10. Solve the initial value problem.

$$x\frac{dy}{dx} = 3x^2 - 2y$$
, $y(1) = 2$

$$y = \underline{\hspace{1cm}} (10)$$

- PART II. Show ALL your work and justify your answer. Each problem is worth 15 points. Only the clearly laid out steps of solving the problem will be graded.
 - 11. Evaluate the integral by making an appropriate change of variables.

$$\iint_{R} \left[\cos \left(\frac{y-x}{y+x} \right) \right]^{2} dA$$

where R is the trapezoidal region with vertices (2,0),(3,0),(0,3), and (0,2).

12. Sketch the graph of the function $f(x) = x^{2/3}(12-x)^{1/3}$.

Label the following objectives on your graph: (a) Asymptotes (b) Local extrema (points and values) (c) Intervals of increase/decrease (d) Concave up/down intervals.

試題隨卷繳回