CALCULUS A

1. Let $f(x)$ be a continuous function on the interval $[0,1]$ and $f(0) = f(1)$. Show that, for any integer $n \geq 2$, there exists some $a \in [0, 1-\frac{1}{n}]$ such that $f(a) = f(a+\frac{1}{n})$. (12pts)

2. Show that $\sin x > \frac{2x}{\pi}$ for all $x \in (0, \frac{\pi}{2})$. (12pts)

3. Find the local and absolute extreme values of $f(x) = x^{\frac{1}{3}}(1-x)^{\frac{2}{3}}$. Sketch the graph of f and indicate its inflection points and asymptotes. (12pts)

4. Find the centroid of a sector of radius a and angular width 2α. (12pts)

5. Derive the recursion formula for the indefinite integral $\int \frac{dx}{(1+x^2)^n}$ and evaluate the improper integral $\int_0^\infty \frac{dx}{(1+x^2)^n}$ for each positive integer n. (16pts)

6. Let $f(x) = \ln(x + \sqrt{1+x^2})$. Find its n-th derivative $f^{(n)}(0)$ at $x = 0$. (12pts)

7. Find the area of the part of the sphere $x^2 + y^2 + z^2 = 4a^2$ that lies inside the cylinder $x^2 + y^2 = 2ay$, where $a > 0$. (12pts)

8. Evaluate the line integral $\int_C ye^x dx + (x^2 + e^x)dy + z^2 e^z dz$, where C is the curve $\mathbf{r}(t) = (1+\cos t)\mathbf{i} + (1+\sin t)\mathbf{j} + (1-\cos t - \sin t)\mathbf{k}$ for $0 \leq t \leq 2\pi$. (12pts)

試題隨卷繳回