※ 注意：請於答案卷上依序作答，並應註明作答之大題及其題號。

1. Find the integral $\int_0^2 \dfrac{12x^2}{\sqrt{16-x^2}}\, dx$. (15%)

2. Let $f(x) = \sum_{n=1}^{+\infty} \dfrac{n}{4^n} x^n$ for any $|x| < 4$.

    Find the function $f(x)$ and the value $f(2)$. (15%)

3. Let $D = \{ (x, y, z) \mid x^2 + y^2 + z^2 = 50 \}$ and the temperature at

    the point (x, y, z) on the D is $T(x, y, z) = 12xz + 16yz$
    Find the highest and lowest temperatures. (20%)

4. Let $f(x)$ be a function such that
    (1) $f(x)$ is continuous on $[0, 1]$, $f(0) = 0$ and $f(1) = 7$.
    (2) $f'(x)$ and $f''(x)$ exist for any $0 < x < 1$.
    (3) $f''(x) > 0$ for any $0 < x < 1$.
    Prove that $f(x) < 7x$ for any $0 < x < 1$. (15%)
    [Hint : there is $0 < \alpha < 1$ such that $f'(\alpha) = 7$.]

5. Let D be the region bounded by the curve $C: \begin{cases} x = t^3 + t^2 \\ y = t^2 + t \end{cases}$ for $0 \le t \le 1$ and

    the line $y = x$. Find the area of D. (15%)

6. Let $\vec{F}(x, y, z) = xy^2 \vec{i} + yz^2 \vec{j} + zx^2 \vec{k}$ be a vector field and

    $S = \{ (x, y, z) \mid x^2 + y^2 + z^2 = 25 \}$ be a surface with outward orientation.

    Find the flux $\iint\limits_S \vec{F} \cdot \vec{N} dS$ of $\vec{F}(x, y, z)$. (20%)

試題隨卷繳回