1. **(24%)** Solve the following initial value problems where the prime sign indicates differentiation with respect to time t.

   (a) $y' + y = (2\cos t)H(t-\pi),\ y(0) = 1$ in which H is a Heaviside unit step function. (6%)

   (b) $y'' + 3y' + 2y = 0,\ y(0) = 1,\ y'(0) = 0.$ (6%)

   (c) $y'' + 3y' + 2y = \delta(t),\ y(0) = y'(0) = 0$ in which δ is a Dirac delta function. (6%)

   (d) $y'' + 3y' + 2y = \dfrac{1}{1+e^t},\ y(0) = 1,\ y'(0) = 0.$ (6%)

2. **(10%)** If $f(x) = |x|,\ -1 \le x \le 1$, find and write out explicitly the beginning (or lower) four terms of its Fourier series.

3. **(17%)** Transform a third-order linear differential equation $y''' + 2y'' - y' - 2y = 0$ into a system of first-order simultaneous linear differential equations, and use only the matrix method to solve the problem.

4. **(16%)** Use Laplace transform to solve the following partial differential equation of vibration:

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2},\ x > 0 \text{ and } t > 0$$
$$u(x,0) = 0,\ u_t(x,0) = b,\ x > 0$$
$$u(0,t) = t,\ t > 0.$$

5. **(15%)** Evaluate the integral

$$\frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} \frac{e^{\sigma t}}{\sqrt{\sigma}}\, d\sigma,$$

   where i is the imaginary unit and the value of γ is some constant. Show all details of your derivations.

   [*Hint*: $\Gamma(x) = \int_0^\infty e^{-\tau}\tau^{x-1}d\tau$ and $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.]

6. **(18%)** Using the following relation, namely

$$\nabla \cdot \mathbf{B} = \lim_{\Delta V \to 0} \frac{\oint_A \mathbf{B} \cdot d\mathbf{A}}{\Delta V},$$

   where A is the surface enclosing the volume ΔV, expand and explicitly write out $\nabla \cdot \mathbf{B}$ in spherical $r-\theta-\phi$ coordinates. Note that $\mathbf{B} = B_r\mathbf{i}_r + B_\theta\mathbf{i}_\theta + B_\phi\mathbf{i}_\phi$. Show all details to get full credit.