

題號： 415

國立臺灣大學 108 學年度碩士班招生考試試題

科目： 近代物理學(B)

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Planck's constant = 6.626×10^{-34} J·s

Electron charge = 1.602×10^{-19} C

Mass of electron = 9.1×10^{-31} kg

Mass of proton = 1.67×10^{-27} kg

Speed of light = 3×10^8 m/s

(7% total)

1. A photon of energy E is scattered by a particle of rest energy E_0 . Find the maximum kinetic energy of the recoiling particle in terms of E and E_0 .

(10% total)

2. Find the wavelength, phase and group velocities of the de Broglie waves of an electron whose kinetic energy is 500 keV. Is the relativistic calculation needed?

(8% total)

3. An electron is in the ground state of a one-dimensional infinite well with length L . Calculate the standard deviation of momentum

$\sigma_p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$, where $\langle p \rangle$ and $\langle p^2 \rangle$ are the expectation values of p (i.e. momentum) and p^2 , respectively.

(10% total)

4. The wave function for a 2p electron in the hydrogen atom is: $\psi(r, \theta, \phi) = R(r)\Theta(\theta)\Phi(\phi)$ where $R(r) = \frac{1}{2\sqrt{6}} \frac{r}{a_0^{3/2}} e^{-r/2a_0}$,

$\Theta(\theta) = \frac{\sqrt{6}}{2} \cos\theta$, $\Phi(\phi) = \frac{1}{\sqrt{2\pi}}$ and a_0 is the radius of the innermost orbit. Find the most probable value of r for finding a 2p

electron.

(12 % total, 4 % for each sub-question)

5. A Hydrogen molecule consists of two Hydrogen atoms separated by a distance $R=2r_0=0.5 \times 10^{-10}$ m each with a mass of $M_H=1.674 \times 10^{-27}$ kg.

a) What is the moment of inertia of the molecule with respect to the point half way between the two atoms? You do not need to provide a numerical answer. Explain the reason why you can consider the atoms as point like objects.

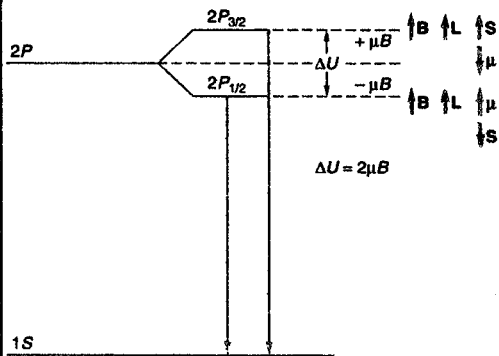
b) Write the kinetic energy in terms of the angular momentum of the molecule. Assuming the angular momentum is an integer multiple of \hbar , express the rotation energy level of the Hydrogen molecule

c) What is the wavelength corresponding to the lowest energy transition?

(8 %)

6. The fine-structure splitting of the $2P_{3/2}$ and $2P_{1/2}$ levels in hydrogen is 4.5×10^{-5} eV, as shown in the drawing below. From this, estimate the magnetic field that the 2p electron in hydrogen experiences. Assume \mathbf{B} is parallel to the z axis. The Bohr magneton constant in this structure is 9.27×10^{-5} J/T.

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(20 % total)

7.

- a) What are the advantages of a four-level laser over a three-level laser? (4 %)
- b) Why is helium needed in a helium-neon laser? Why not just use neon? (4 %)
- c) Compare the critical population inversion necessary for laser action in the ruby and He-Ne lasers. Compute the corresponding power requirements and use the table below. (8 %)

Parameter	Ruby laser	He-Ne laser
λ	694.3 nm	632.8 nm
f	$4.32 \times 10^{14} \text{ s}^{-1}$	$4.74 \times 10^{14} \text{ s}^{-1}$
n (refractive index)	1.76	1.00
t_s	$3 \times 10^{-3} \text{ s}$	10^{-7} s
t_p	$2.9 \times 10^{-8} \text{ s}$	$3.3 \times 10^{-7} \text{ s}$
Δf	$3.3 \times 10^{11} \text{ s}^{-1}$	$9 \times 10^8 \text{ s}^{-1}$
N (Cr^{3+} concentration)	$2 \times 10^{19} / \text{cm}^3$	—

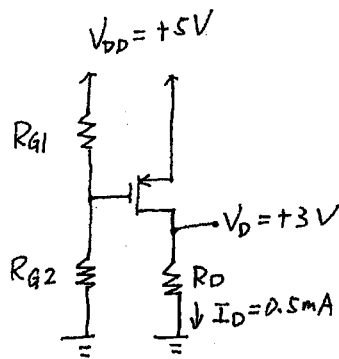
- d) Based on discussions and calculations above, please give four advantages of He-Ne laser over Ruby laser. (4 %)

(15% total)

8. (MOS) Design the circuit below so that the transistor operates in saturation with $I_D=0.5\text{mA}$ and $V_D=+3\text{V}$. Let the enhancement-type PMOS transistor have $V_t=-1\text{V}$ and $k_p'(W/L)=1\text{mA/V}^2$. Neglect the channel length modulation effect ($\lambda=0$).

(a) What is channel length modulation effect? (5%)

(b) What is the largest value that R_D can have while maintaining saturation region operation? (10%)



(10% total)

9. (Diode) Diodes can be used to realize rectifiers, which are commonly used in power circuits.

- (a) Please draw circuit diagram of a "half-wave rectifier". (5%) (Hint : only power source, diodes and load resistors are used)
- (b) Please draw the input and output waveforms in the time domain (voltage vs time), assuming the diode resistance is much smaller than the load resistance, and a sinusoidal input waveform. (5%)