1. (15 points) For all positive integers n, compute

$$\sum_{k=0}^{n}(k3^{k-1}\cdot C(n,k)),$$

where $C(n,k)$ is the coefficient of the x^k term in the expansion of $(1+x)^n$

2. (15 points) Solve the following recurrence:

$$\begin{aligned} a_1 &= 3, \\ (1-\frac{1}{n})a_n &= \frac{n}{n-1}a_{n-1}+1, \text{ for all } n\geq 2. \end{aligned}$$

3. (10 points) Let n be a positive integer such that $n^5+(n+1)^5 \equiv 0 \pmod{25}$. Find all possible values for $(n+2)^5$ mod 10. Show your derivations. You must briefly justify that no other values are possible.

4. (35 points) For each of the following statements, determine whether it is true of false. No explanation is needed. You get +5 points for every correct answer and -6 points for every incorrect one. (0 points if you do not answer.)

   (a) $\neg\Big(\exists x(P(x)\wedge Q(x))\Big) \equiv \forall x(P(x)\rightarrow \neg Q(x))$.

   (b) In propositional logic, $\{\oplus, \rightarrow\}$ is a functionally complete set.

   (c) Given five propositional logic statements using a single variable p, there exists two of them which are equivalent to each other.

   (d) There exists an injection from $\mathbb{Q}$ to $\mathbb{R}$.

   (e) If A and B are two uncountable sets, then $|A|=|B|$.

   (f) If a relation R is symmetric, then R^3 is also symmetric.

   (g) If R is a partial ordering on a finite set A, then R has at least one maximal element.

5. (15 points) Let $G=(V,E)$ be a simple planar undirected graph with at least 3 edges. All simple cycles in G have length at least 5. Is it always true that $3|E|\leq 5|V|-10$? Prove your answer formally.

6. (10 points) Let G be a connected undirected graph with 15 vertices. If G is not planar but removing any edge from G results in a planar graph, how many edges does G have? List all possible answers and prove the correctness of your answer.