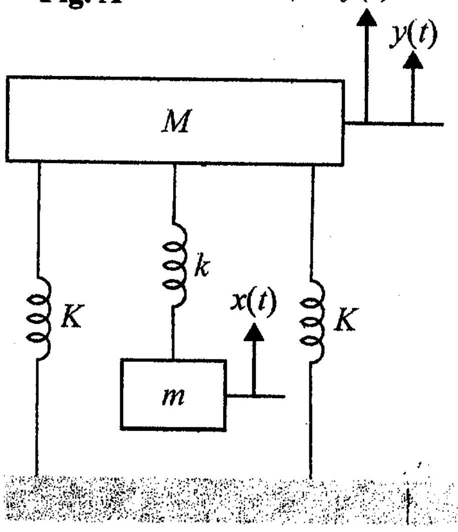
請於答案卷上作答，於試題卷上作答者，不予計分。

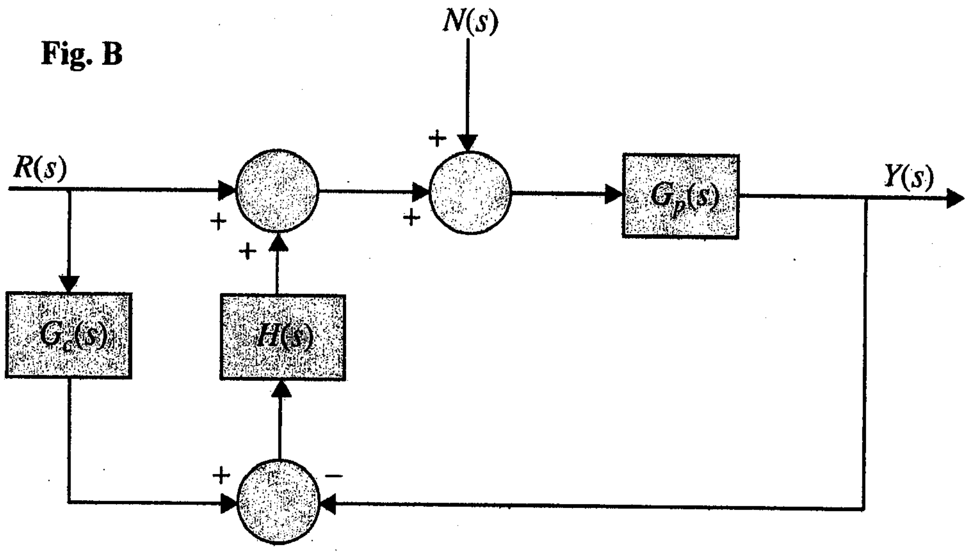
1. Vibration absorbers are used to protect machines that work at the constant speed from steady-state harmonic disturbance. **Fig. A** shows a simple vibration absorber.
Assuming the harmonic force $f(t) = A\sin(\omega t)$ is the disturbance applied to the mass M:
   (a) Derive the state space of the system. Consider $y(t)$, $y'(t)$, $x(t)$, and $x'(t)$ as the state variables (**in the listed order**). 【計分：8 分】
   (b) Draw a state diagram for the system. 【計分：4 分】
   (c) Determine the transfer function of the system, $Y(s)/F(s)$. 【計分：3 分】

Fig. A



2. **Figure B** shows the block diagram of a control system with conditional feedback. The transfer function $G_p(s)$ denotes the controlled process, and $G_c(s)$ and $H(s)$ are the controller transfer functions.
   (a) Derive the transfer functions $Y(s)/R(s)|_{N=0}$ and $Y(s)/N(s)|_{R=0}$. Find $Y(s)/R(s)|_{N=0}$ when $G_c(s) = G_p(s)$. 【計分：6分】
   (b) Let $G_c(s) = G_p(s) = 100/[(s + 1)(s + 5)]$. Find the output response $y(t)$ when $N(s) = 0$ and $r(t) = u_s(t)$, where $u_s(t)$ is a unit-step input. 【計分：4 分】
   (c) With $G_c(s)$ and $G_p(s)$ as given in part (b), select $H(s)$ among the following choices such that when $n(t) = u_s(t)$ and $r(t) = 0$, the steady-state value of $y(t)$ is equal to zero. (There may be more than one answer.)
   $H(s) = \dfrac{1}{s(s+1)}$, $H(s) = \dfrac{10}{(s+1)(s+2)}$, $H(s) = \dfrac{10(s+1)}{(s+2)}$, $H(s) = \dfrac{K}{s^n}$ (n = positive integer) Select n.
   Keep in mind that the poles of the closed-loop function must all be in the left-half s-plane for the final-value theorem to be valid. 【計分：10 分】

Fig. B



3. Consider the following state equation and output equation:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 6 & -5\\ 1 & 0 & 2\\ 3 & 2 & 4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}2\\ 6\\ 2\end{bmatrix}u;\quad y = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Find the transformation $\mathbf{x}(t) = \mathbf{T}\bar{\mathbf{x}}(t)$ so that the state equations are transformed into the diagonal canonical form (DCF) if **A** has distinct eigenvalues 【計分:9 分】 and Jordan canonical form (JCF) if **A** has at least one multiple-order eigenvalue. 【計分：6 分】

4. Consider a unit mass system with transfer function $G(s) = \dfrac{1}{s^2}$ (in meters per Newton), please determine the forward controller K_f and the parameters K_p and K_d in feedback controller so that the output of the

closed-loop system to unity step reference input (as shown in **Fig. C**) satisfies the specifications: rising time is less than 0.01second, overshoot is less than 5% and steady state is less than 1% **【計分：10 分】**

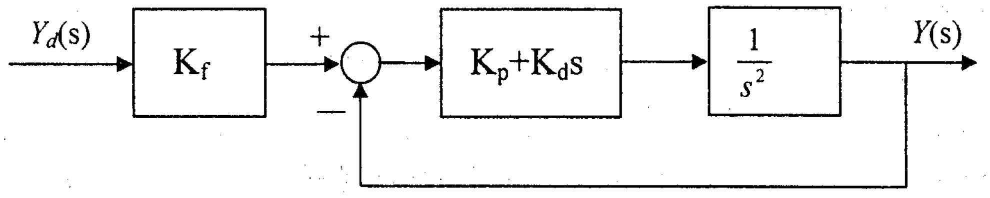


Fig. C (figure for problem 4)

5. We wish to design a velocity control for a servomechanism. The transfer function from current to velocity (in millimeters per millisecond per ampere) is $\frac{V(s)}{I(s)}=\frac{15(s^2+0.9s+0.8)}{(s+1)(s^2+1.1s+1)}$. We wish to design a type I feedback system so that the transient satisfies the rising time is less than 4 millisecond, setting time is less than 15 millisecond, overshoot is less than 5%.
   (a) Use the compensator K/s to achieve type I behavior, and sketch the root-locus with respect to K. **【計分：10 分】**
   (b) Show on the plot the region of acceptable pole locations corresponding to the specifications. **【計分：10 分】**

6. The Nyquist diagram for a stable, open loop system is sketched in **Fig. D**. The proposed operating gain is indicated as K_0, and arrows indicate increasing frequency. Give your best estimates of the following quantities for the closed-loop (unity feedback) system
   (a) phase margin, (b) damping ratio, (c) closed-loop bandwidth, (d) range of gain for stability (if any), (e) system type (0, I, or II) **【計分：20 分】**

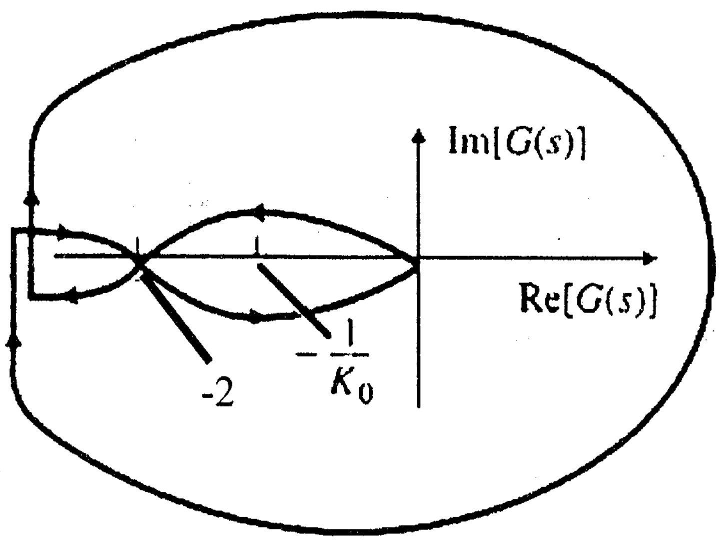


Fig. D (figure for problem 6)

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