※ 注意：請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。

※ n refers to the number of elements in the data structure unless told otherwise.
※ In the data structure, numerical values refer to keys unless told otherwise.

一、是非題(共 15 小題，每題 4 分，答對得 4 分，答錯倒扣 4 分，至多扣到該大題零分為止。)
命題正確請填(A)，錯誤請填(B)

1. In a circular doubly linked list, deleting a node other than the head needs to modify at least 4 links.

2. It is possible that the post-order traversal of a BST yields 1, 2, 3, 4, 5, 6, 7, 8, 9, where the numbers are keys.

3. Constructing a min heap with n elements requires $\Theta(n\log n)$ time.

4. For a streaming data where numbers are given one-by-one, keeping track of the median requires $\Theta(n)$ time.

Answer questions 5 to 8 for the following setting.
Let s(node) be the number of comparisons required to find the node in the BST, e.g. s(root) = 1. Let t(node) be the number of the nodes in its left subtree, e.g. t(leaf) = 0.

5. The summation of s for all nodes can be as large as $\Omega(n^2 \log n)$.

6. The summation of s for all nodes can be as small as $O(n \log n)$.

7. With the information of t(node), the median can be found in $O(\log n)$ time.

8. If we want to maintain $s()$ and $t()$ for all nodes on a balanced BST such as AVL or red-black tree, insertion requires $\Theta(n)$ time.

9. If we use rotations to transfer a BST into an AVL-tree, at most $O(\log n)$ rotations are required.

10. In an AVL tree, every subtree is also an AVL tree.

11. In a balanced BST such as AVL or red-black tree, finding the element with the 2nd smallest key requires $\Theta(n)$ time.

12. In a red-black tree, there can be at most 2 red nodes in any path.

13. In a red-black tree, every subtree is also a red-black tree.

14. Consider the following implementation of extendable array. The array starts with some small constant size with no element. It supports two operations: insert and delete. The insertion inserts an element at the end (suppose there are 4 elements in the array; the new element is inserted at the 5th position). When the array is full with n elements, it allocates a new empty array with $2n$ slots, copies all n elements to the first n slots of newly allocated area, and releases the original n slots. The deletion deletes an element at the end (suppose there are 6 elements; the 6th element is erased). When the array is half empty with n elements, it allocates a new empty array with n slots, copies all n elements to the newly allocated area, and releases the

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original $2n$ slots. The following statement is True or False? Starting with empty, n insertions and n deletions in arbitrary order takes $O(n)$ time in the worst case.

15. Concerning open addressing techniques for the hash table, linear probing suffers from the primary clustering, while quadratic probing suffers from the secondary clustering.

二、複選題(每題 10 分 5 個選項，其中至少有一個是正確的答案。各題獨立計分，每答對一個選項，得 2 分；每答錯一個選項時，倒扣 2 分。每題倒扣至零分為止。不作答者，得零分。)

16. Which of the following trees achieve $O(\log n)$ in time in the worst case for both insertion and deletion, where n is the number of elements stored in the tree?
    (a) AVL trees
    (b) Red-black trees
    (c) 2-3 trees
    (d) 2-3-4 trees
    (e) Splay trees

17. Consider an undirected graph G=(V,E). Which of the following statement(s) is(are) true?
    (a) Finding all connected components can be done in O(|V|+|E|) time if the adjacency matrix is provided.
    (b) Finding all connected components can be done in O(|V|+|E|) time if the adjacency list is provided.
    (c) If edges are dynamically added into the graph one-by-one, keeping track of all connected components for each edge addition can be done in O(log|V|) time.
    (d) Detecting whether the graph is cyclic can be done in O(|V|+|E|) time if the adjacency list is provided.
    (e) None of the above.

18. Which of the following statement(s) of **undirected** graph is(are) a tree?
    (a) A graph with n vertices and n-1 edges.
    (b) A bipartite graph that is also planar.
    (c) A clique that is also planar.
    (d) A graph without any clique.
    (e) None of the above.

19. Consider the spare matrix of size n by n and n non-zero entries. We store the non-zero entries by a linked list per row (list-per-row representation). Specifically, the whole matrix is stored in a one dimensional array containing linked lists, where lists are indexed by the column number. Which of the following statement(s) about matrix multiplication is(are) true?
    (a) The running time for the best case is $\Theta(n)$
    (b) The running time for the best case is $\Theta(n\log n)$
    (c) The running time for the worst case is $\Theta(n \log n)$
    (d) The running time for the worst case is $\Theta(n^2)$
    (e) The running time for the worst case is $\Theta(n^2 \log n)$

試題隨卷繳回