

Problem 1. (25%)

Assume steady-state laminar flow between large flat parallel plates as shown in figure (A).

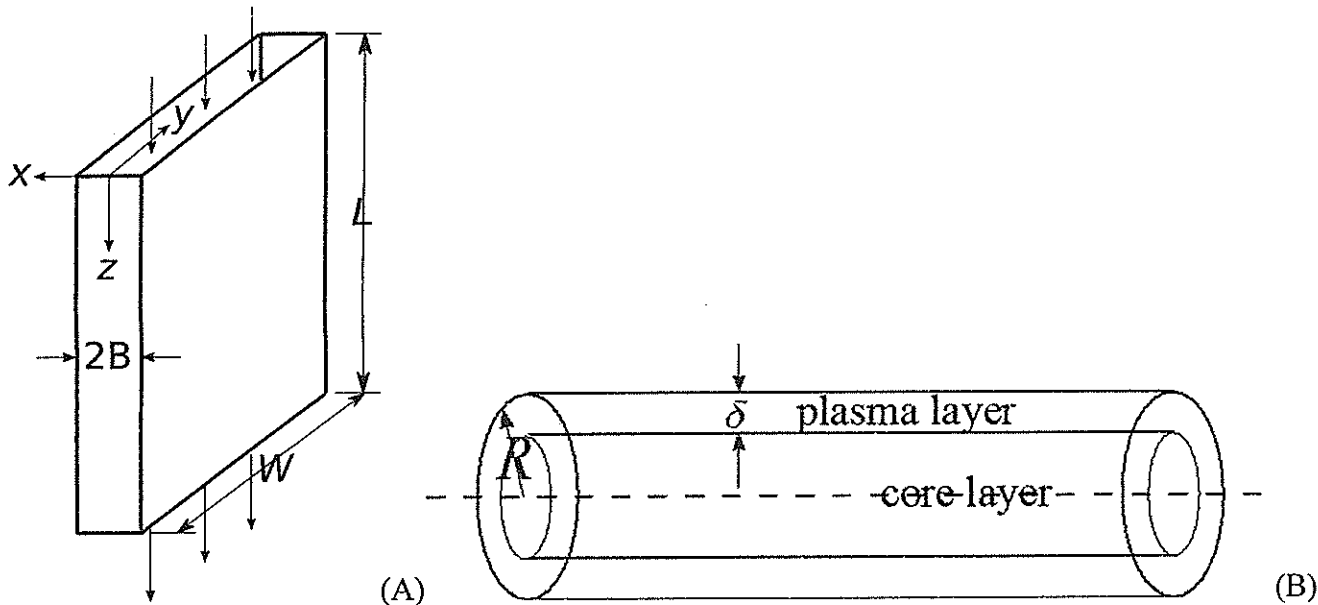
- (a) Derive the velocity profile between the plates for Newtonian fluid. (12%)
- (b) Both plates are maintained at constant temperature T_0 . Take into account explicitly the heat generated by viscous dissipation and neglect the temperature dependence of μ and k . Derive the expression for the temperature distribution $T(x)$ between the plates. (13%)

Hint: Equation of energy for pure Newtonian fluid:

$$\rho C_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] + \mu \Phi_v$$

The dissipation function for Newtonian fluids:

$$\Phi_v = 2 \left[\left(\frac{\partial v_x}{\partial x} \right)^2 + \left(\frac{\partial v_y}{\partial y} \right)^2 + \left(\frac{\partial v_z}{\partial z} \right)^2 \right] + \left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right]^2 + \left[\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} \right]^2 + \left[\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right]^2 - \frac{2}{3} \left[\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right]^2$$



Problem 2. (25%)

The blood flow within a tube or some other vessel is divided into two regions (as shown in figure (B)): a central core that contains cells with a viscosity of μ_c and a cell-free plasma peripheral layer with thickness δ and a viscosity of plasma denoted by μ_p . In each region, the flow is considered Newtonian and at steady state. Derive the discharge rate of blood for both plasma and core layers. (25%)

Problem 3. (15%)

- (a) The molar flux J for one-dimensional diffusion (in x direction) of molecular A in water can be expressed as a function of C_A and x as the following equation. What will the equation be if three-dimensional diffusion is considered? (4%)

$$J = -D \frac{dC_A}{dx}$$

- (b) Apply the shell balance technique to show that the diffusion equation for molecular A in water can be expressed as below if there is a homogeneous second-order reaction $A \rightarrow B$ accompanying the one-dimensional diffusion of A along the x axis. (7%)

$$\frac{\partial C_A}{\partial t} = D \frac{\partial^2 C_A}{\partial x^2} - k C_A^2$$

- (c) Is it possible to exist a steady-state concentration profile $C_A(x) \neq f_n(t)$ in case (b)? Why? (4%)

Problem 4. (20%)

In electrochemistry, a limiting current is referred to as a reaction current that is governed by the diffusion control only and cannot be further enlarged by increasing the electrode potential. It is well-known that the limiting current (i_L) for a generalized redox reaction $O_{(aq)} + ne^- \rightarrow R_{(aq)}$ occurred at a planar inert electrode (e.g., Pt) can be described the Cottrell equation as below, where n , F , A , D_O , C_O^* , and t represent the number of electron transfer, Faraday's constant, electrode area, diffusion coefficient of species O, bulk concentration of O (the concentration far from the electrode surface), and time, respectively.

$$i_L = \frac{nFAD_O^{1/2}C_O^*}{\pi^{1/2}t^{1/2}}$$

- (a) Design an experiment to determine the diffusion coefficient of Cu^{2+} in water. (5%)
 (b) Show that the limiting current (i_L) and molar flux of O (J_O) at the electrode surface ($x = 0$) can be correlated with each other by the following equation. (5%)

$$i_L = -nFAJ_O(x = 0)$$

- (c) Describe the steps to derive the Cottrell equation from Fick's laws. (There is no need to derive the Cottrell equation.) (5%)
 (d) Why i_L is proportional to $t^{-1/2}$ instead of t^{-1} ? (5%)

Problem 5. (15%)

Human insulin (used for Type I diabetes treatment) can be produced by recombinant DNA technology, in which a human insulin gene is inserted and expressed (i.e., to produce insulin) in *E. coli* (gut bacterium) cells. It has become a common laboratory practice to obtain insulin by the following process steps: (i) Growth (amplification) of *E. coli* cells with the insulin gene in a liquid culture medium at 37 °C overnight. (ii) Harvest (collection) of *E. coli* cells by separating the cells (as pellets) from the liquid medium. (iii) Extraction of cell lysates (cytosolic materials containing total proteins, nucleic acids and salts) from *E. coli* cells by "dissolving" (with mild detergent) or "breaking" (with a shear stress force) the cell membrane. (iv) Separation of total proteins from nucleic acids in cell lysates by $(NH_4)_2SO_4$ precipitation, in which total proteins are precipitated in the presence of concentrated $(NH_4)_2SO_4$. (v) Purification of insulin by flowing the total protein mixture (dissolved in a buffer solution) through an antibody-capturing affinity column followed by a recovery elution of insulin. (vi) Lyophilization (freeze-drying*) of the purified insulin solution into dry powders. (*Freeze-drying works by freezing the aqueous protein sample and then reducing the surrounding pressure to allow the frozen water in the sample to sublime directly from the solid phase to the gas phase until only dry protein powders are left.)

Now, design TWO operational units to tackle any two steps in the above process for mass production of insulin. (People often assume that a chemical engineer is good at doing such a job.) Please draw an illustrative scheme for each of your operational unit and describe how it works briefly with the knowledge of chemical engineering. (15%)