※ 注意：請於試卷內之「非選擇題作答區」作答，並應註明作答之題號。

1. (25%) A horizontal rod with length $L = 10$ m and cross-sectional area $A = 10$ cm $\times 10$ cm $= 0.01$ m^2 is made of a homogeneous material with Young's modulus $E = 1$ GPa such that the axial rigidity is $EA = 10$ MN. The rod being fixed at the two ends $x = 0$ and $x = 10$ m is acted upon by a right-directed axial load $p(x) = 0.2$ kN/m uniformly distributed over the left-half span $0 \leq x \leq 5$ m and also by a concentrated right-directed axial load $P = 1$ kN applied at the mid-span $x = 5$ m.

   (a) (8%) Find the reactions at the two ends. Find (and plot the diagrams of) the axial force $N(x)$.

   (b) (8%) Find (and plot the diagram of) the axial displacement $u(x)$.

   (c) (9%) Compute the total external work W. Find (and plot the diagrams of) the stored energy per unit length $U_\ell(x)$, and then integrate to obtain the total stored energy U. Is $W = U$? Why?

2. (20%) Let ϵ be the axial strain corresponding to the axial force N, and κ the curvature corresponding to the bending moment M. A rectangular section of width b and depth h is composite with two materials, Young's moduli $E_1 = E$ in the upper half and $E_2 = 2E$ in the lower half. Locate the modulus-weighted centroid of the composite section and then derive formulae and draw diagrams showing the normal strain and stress distributions over the cross section in terms of $\epsilon, N, \kappa, M, b, h, E$.

3. (20%) A simply supported bar (beam/shaft) with channel section of equal flanges is subjected to transverse loads. The equal-flange channel member is known to have one plane of symmetry.

   (a) (10%) Plot diagrams showing the distributions (magnitudes and directions) of the normal and shear stresses when the channel is so placed that its plane of symmetry coincides with the loading plane of the transverse loads. Are there shear stresses due to torsion?

   (b) (10%) Plot diagrams showing the distributions (magnitudes and directions) of the normal and shear stresses when the channel is so placed that its plane of symmetry is perpendicular to the loading plane of the transverse loads. Are there shear stresses due to torsion?

4. (15%) Two thin-walled pressure vessels, one spherical and the other cylindrical, with equal inner radius $r = 2$ m are both subjected to internal pressure $p = 20$ MPa. The vessels are made of the same material with Young's modulus $E = 520$ GPa, the shear modulus $G = 200$ GPa, and the allowable shear stress $\tau_{allow} = 810$ MPa. Determine the thicknesses required for the two vessels.

5. (20%) Let $N(x)$ be the axial force, $V_y(x), V_z(x)$ the shear forces, $M_y(x), M_z(x)$ the bending moments at the cross section x $(0 \leq x \leq L)$ of a prismatic member (rod/beam/column) of length L with axial rigidity EA and bending rigidities EI_y, EI_z, which is subjected to a pair of compressive load P at both ends.

   (a) (6%) Formulate the differential equations of force/moment equilibrium for an engineering ("Euler-Bernoulli") rod/beam.

   (b) (8%) Formulate the differential equations of force/moment equilibrium for a buckling column.

   (c) (6%) Point out the main difference between them.

試題隨卷繳回